VERIFICATION OF THE VALIDITY OF DARCY'S LAW FOR UNSATURATED MEDIA USING FINITE DIFFERENCE METHOD

by

SHARAD NAGAICH, B.E.

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CHAPTER 1

INTRODUCTION

The rapid expansion of population, industry, and agriculture in arid regions has brought about a substantial increase in usage of ground water resources to supplement surface water supplies. Ground waters and surface waters are not separate and independent units, as is often assumed, but are closely interrelated. Because of their profound effects on the behavior of surface waters, ground water development should be undertaken after a careful planning effort which includes an analysis based on a thorough understanding of ground water movement. When carefully planned, ground water projects can result in more efficient and benefecial utilization of available water resources. Poorly planned projects can be determental to the environment, to the users of surface water, and to the overall efficiency of water resources utilization.

Ground water accounts for a major portion of the world's fresh water resources; i.e., 0.6 percent of the world's total fresh water. As much of the ground water below a depth of 0.5 mile is saline or costs too much to develop with present technology and economic conditions, the total volume of readily usable groundwater has been estimated as 5.2×10^6 million acre-feet. This is much more than the 1.5×10^5 million acre-feet of the fresh water stored in the lakes and streams. Next to glaciers and ice caps, groundwater reservoirs are the largest storage basins for fresh water in the world's hydrological cycle. Ground water in the unsaturated zone (vadose zone) accounts for about 0.005 percent of the total available water on the earth (Bower, 1978).

Increasing use of ground water from major aquifers in the United States has required a better understanding of gravity drainage in soil strata. Most of the processes involving soil-water interactions in the field, and particularly the flow of the water in the root zone of most crop plants, occur while the soil is in an unsaturated condition. The formulation and solution of unsaturated flow problems very often require the use of indirect methods of anlysis, based on approximations or numerical techniques. In recent decades, however, unsaturated flow has become one of the most important and active topics of research in ground water flow problems, and this research has resulted in significant theoretical and practical advances. Recovery of water from an initially saturated porous soil depends not only on the architecture of the pore space and the nature of the minerals constituting the soil strata but also on the process used to recover the water.

The unsaturated zone is the medium through which pollutants move from the soil surface to ground water. Pollutant substances are subjected to complex physical, chemical and biological transformations while moving through the unsaturated zone and their displacement depends upon the transport properties of the water-air-porous media system. The movement of pollutants in unsaturated soil is based on Richard's equation which has been widely accepted to be a modified form of Darcy's equation. Pollution caused by human activities, agriculture, and industry has brought about a growing interest in the role of the unsaturated zone in ground water pollution. Due to the complexity and multidisciplinary nature of the subject, this zone is being investigated by the specialists from various scientific disciplines such as soil physicists, chemists, and biologists, and environmental engineers. The physical properties of the soil water make the process more complicated. Whenever an attempt is made to drain a given strata of its water, mechanisms which retain the water become operative, and generally only a part of the water can be withdrawn.

The retention mechanism depends largely on the process used to drain the media. For example, if fully saturated loose sand is freed of its stored water by heating to a temperature just above the boiling point, recovery will be 100 percent; all the water will be vapourized and may be collected. If, however, the sand is drained by gravity, capillary retention becomes operative and only a part of the water may be recovered. Water may be removed by heating a saturated clay, and it will be not only absorbed water but perhaps water of combination, according to the temperature. The clay may even be changed irreversibly. If, however, gravity is used to drain the clay, ordinarily little or no water can be recovered (Smith, 1961).

In this research an effort was made to establish the validity of Darcy's law for unsaturated soil media. This is necessary because such a study has not been previously attempted for long columns of sand viz. 30 feet high under constant environmental conditions. The height of the column has been chosen to be 30 feet because it exceeds the net positive suction head of water at this location; i.e., water can not be lifted 30 feet at Lubbock, Texas (3254 feet MSL, approximately). The nonlinearity of the differential equation of motion needs to be examined further for the above conditions.

The specific objectives of this research are to :

1) Collect water content and pressure potential data using a 30 foot long sand column;

2) Develop a mathematical model describing the movement of water in the unsaturated soil media using the finite difference method;

3) Develope a computer code to solve the mathematical model using numerical methods;

4) To predict the time required for the sand column to cease draining; and,

5) Examine the validity of Darcy's law for the movement of water in the unsaturated soil media under constant environmental conditions.

The validity of Darcy's law can be tested by using either steady-state or transient state flow systems. Flux, gradient and water content in steady state systems are constant in time, while they vary in transient state flow systems. The results from a steady state flow system depend on the distribution of matric potential and Darcy's law makes a definite prediction about that. Measurements based on a steady state conditions are more convenient to carry out and often more accurate. The difficulty, however, lies in setting up the system, which may take a long time to stabilize. A transient flow system depends on the agreement of the water content versus time relationships predicted by simulation models and experimental values. (Hillel, 1982)

This report includes a review of pertinent literature which is presented in the next chapter. An effort has been made to review Darcy's law and its applicability. Chapter Three is devoted to a description of the experimental set-up and procedures adopted to collect the necessary data. Chapter Four describes the derivation of the theoretical model and the computer code. Results of the study are discussed in the Chapter Five. Lastly, conclusions and recommendations are made in the last Chapter.

CHAPTER II

LITERATURE REVIEW

In the year 1856, Henry Darcy described in an appendix to his book, Les Fontains Publiques de la Ville de Dijon (King, 1899), a series of experiments on the downward flow of water through filter sands, whereby it was established that the rate of flow is given by the following equation:

$$q = - K \left(\frac{h_2 - h_1}{L}\right) \quad . \tag{2.1}$$

The volume of water crossing a unit area in unit time is q, L is the thickness of the sand, h_1 and h_2 are the heights above a reference level of the water in a manometer terminated above and below the sand layer respectively, and K is a factor of proportionality called soil permeability or conductivity. This relationship, appropriately, soon became known as Darcy's law.

Early Investigations

Darcy's law, though originally conceived for saturated flow only, was extended by Richards (1931) to unsaturated flow, with the provision that the conductivity is now a function of the matrix suction head, (K(h)):

$$q = -K(h)\nabla H \tag{2.2}$$

where H is the hydraulic gradient, a term which includes both pressure and gravitational components.

In processes involving both wetting and drying phases, the fuction K(h) is highly hysteretic. However, the relationship between conductivity and volumetric water content K(θ), or the degree of saturation K(S), is affected to a much lesser degree by past wetting and drying history than is the K(h) function. Thus Darcy's law for unsaturated soil can also be written as:

$$q = -K(\theta)\nabla H \quad . \tag{2.3}$$

This form however, leaves the problem of dealing with the hysteresis between h and θ unresolved.

In the past, the term "capillary conductivity" was used to indicate the conductivity of unsaturated materials (Richard, 1931). It was reasoned that conductivity of an unsaturated material was not a constant as it is under saturated conditions. At the present, the term "hydraulic conductivity" for both saturated and unsaturated flow is accepted.

Stearns and King (1942) concluded that, at low pressure gradients, Darcy's law is in exact agreement with experiments and is entirely analogous to the law of

Poiseuille for the flow of liquids through capillary tubes. However, both of these laws fail to hold for high pressure gradients and high fluid velocities. During the past, studies have been concerned with the nature of the deviation which occurs at large hydraulic gradients. When the hydraulic gradient exceeds a critical value, the flow velocity is no longer proportional to the hydraulic gradient, but increases more rapidly than the gradient. This relationship could be explained if viscous forces no longer mask the inertia and turbulant forces, then not all the driving force of the hydraulic gradient is used to overcome viscous resistance. Compared with the extensive study conducted on deviations under large gradients, much less work has been directed towards testing of Darcy's law at low gradients.

Scheidegger (1957) recognized the possibility of deviations arising from a so-called "boundary effect" from ions in solution, and from non-Newtonian fluids.

According to Swartzendruber (1962), there are at least three reasons for reconsidering the question of the validity of Darcy's law. Firstly, there is the recent appearance of data which simply do not obey Darcy's law, at least from a phenomenological standpoint represented by velocity-gradient curve. Secondly, there is increasing evidence that water properties near clay surfaces are not the same as those of normal bulk water. Lastly, the work of King (1898) who considered such deviations more than seventy years ago, seems largely to have been ignored.

Swartzendruber (1963) proposed a modified velocity gradient relationship which did fit the experimental data, and which contained Darcy's proportionality as a special case. He concluded that if deviations could be caused by clay-water interactions in water-saturated porous media, similar deviations should occur in unsaturated media. The magnitude of deviations might be greater, since the water must move in closer proximity to the particle surfaces.

The idea of a threshold hydraulic gradient was justified as proposed by Miller and Low (1963). Their proposed equation was found to be suitable for expressing non-Darcy behavior, both with and without threshold gradients. However, the introduction of $D = D(\theta, d\theta/dz)$ into nonlinear diffusion analysis was ruled out because of expected mathematical complications.

In another study conducted by Rawlins and Gardner (1963), the validity of the diffusion equation for unsaturated flow of soil water was tested. The equation can be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\,\frac{\partial \theta}{\partial x}\right) \qquad (2.4)$$

where $D(\theta) = K(\theta)\dfrac{\partial h(\theta)}{\partial \theta}$, and

$D(\theta)$ is diffusivity.

Diffusivity versus water content curves were constructed for flow of water in an horizontal soil column and it was concluded that the diffusivity is not a unique function of water content. However, the effects of the potential function and the hydraulic conductivity function on diffusivity could not be ascertained and further research was envisaged in this regard.

In 1980, Sposito undertook a detailed first principal study to determine the exact equation of a linear momentum balance for water in an unsaturated soil. He showed that an approximate momentum balance equation presented originally by Raats and Klute data can be used to demonstrate unequivocally that the flow of water through a rigid, homogeneous, isotropic, unsaturated soil will obey the Darcy's law within 10^{-12} to 10^{-15} seconds after the gradient in the total potential of soil water has been applied. Also, an exact equation of motion for the Fourier component of the water mass flux density vector was derived using standard methods of non-equilibrium statistical mechanics. The study led to the conclusions that: (a) water mass density and mass flux density vector constitutes a complete set of strongly coupled, macroscopic dynamical variables, and (b) time scale over which these two variable change significantly is much longer than that over which any other dynamical quantities vary. If these two conditions are met then, according to statistical mechanics, the flow of water through unsaturated soil will be described accurately over all macroscopic time intervals by the Darcy's law.

Column Drainage And Model Studies

The drainage of columns of porous media has been the subject of much research, but very few of these studies have determined the moisture distribution in the soil media when the columns are of relatively long length. Studies have also been made on different periods of drainage. Results of a few of these studies are summarized below.

Hazen (1892) presented significant results on tests of the water-retaining and water-yielding capacities of eight different sands used for filtering sewage. These tests basically related specific yield and specific retention to the effective size and uniformity coefficient of the sands as obtained from particle-size analysis.

King (1899) packed five sorted sands of different particle sizes in columns 8 feet in length and 5 inches in diameter. The materials were slowly saturated from below and then drained for two and a half years. Outflow readings were taken during the period of drainage and the moisture content was determined at the end of the drainage period. The apparatus was designed to prevent moisture loss by evaporation. The results of King's tests showed some discrepancy between the porosity and the total water content and gave lower values for specific retention than Hazen's tests, especially for the coarser samples.

In 1911, Green and Ampt defined the importance of three constants i.e., specific pore or interstitial space (S), permeability of water (K_w) or permeability of air (K_a), and capillary coefficient (C). These constants were expected to replace the determination of the sizes of the soil particle in the usual "mechanical analysis" for defining the flow of water and air in the porous media. Values for these terms were constant for a particular soil under a particular conditions; they were, however, all interdependent, and were liable to considerable alteration when the soil was disturbed as by the ordinary farm operations of ploughing, rolling, etc. or by the ordinary process of taking the sample. Moisture content also caused variations in values of the constants.

Experiments by Green and Ampt (1911) were conducted with uniform glass beads of 0.25 to 1.0 mm diameter for defining the effect of specific pore intersticial space. However, for determining the behavior of C and K_w or K_a, three different kinds of soil samples were used. These samples were packed in glass tubes (1 inch diameter and 30 inches in length). The soils were uniformly packed with air-dry loam to within 6 inches of the upper end; the lower end of the tube being closed with a disk of copper gauze supported by a perforated cork. The authors concluded that the movement of air and water through the test soils conformed to the equations connecting the rate of motion with the three constants.

Lebedeff (1927) drained tubes (4 to 5 cms in diameter and 1 to 3 m in length) filled with sandy soils. A head of water of 2 cm was used to saturated the soil. After drainage, the water retained in 10 cm segments of the column was determined gravimetrically and showed that a uniform moisture content prevailed in the upper part of the columns. The author also saturated 6 pairs of tubes that were 10,20,30,40,50, and 100 cm in length. One tube of each pair was immediately sampled for moisture content. The other tubes were allowed to drain for 3.5 days, the moisture content became constant at a height of about 40 cm. Changing the height of the column did not change this distribution; it gave only a longer section of column with constant moisture content. Saturated columns with layers of sand and loess also were drained. More water was retained in the fine textured material when underlain by coarse textured material than when underlain by material of the same type.

Stearns, Robinson, and Taylor (1930) collected undisturbed columns of soil in metal cylinders 18 inches in diameter and 36 inches in length in the Mokelumne area of California. A bottom was soldered on each cylinder and small diameter observation wells were installed in each soil column. A water table was established near the top of the column. Measured volumes of water were then withdrawn and added alternately and water levels were observed. The time intervals for the water levels to reach equilibrium were very short-only one was over 50 minutes. A larger volume of water was collected for a rising water table than for a falling water table.

White (1932) collected undisturbed soils in steel cylinders 12 or 18 inches in diameter and 18, 36, or 54 inches long. With the bottom of the cylinders sealed by means of a steel plate, small wells were sunk into the material enclosed within the cylinders. In these wells the effects on the water level produced by adding or withdrawing measured quantities of water were observed. Equilibrium was not established until 24 hrs after the addition of water and about 48 hrs after its removal. Data indicated the average specific yield for clay and clay loam ranged from about 1 to 7 percent.

Eckis and Gross (1934) packed three 4 by 40 inch cylinders with carefully sorted sand and saturated the columns with a measured quantity of water. Water was then drained from the columns by means of a well attached by a U-tube to the outside bottom of the cylinder. The water table was lowered by slowly removing a measured quantity of water for 1 month, and a period of 30 to 60 days was allowed for the new water table to reach equilibrium. The difference between the water levels was then used to calculate the water drained by gravity. The columns were allowed to drain for an additional 18 months. The cylinders were then split and the moisture contents were determined for 1 inch increments of the column length. Evaporation effects were found at depths as great as 27 inches below the top of the columns. Less than 10 percent of the water retained in the column at the end of the 30 to 60 days period drained out in 18 months.

Columns 18 inches in diameter and at least 42 inches long were used by Piper et al. (1939) to study specific yield in the Mokelumne area in California. Specific yield was determined by measuring the volume of material saturated or unwatered when measured quantities of water were added or withdrawn from the columns of undisturbed materials. Water levels were affected by both temperature and barometric pressure. No less than 21 days were allowed for a new water level to attain equilibrium and some tests continued for as long as 220 days. The longer drainage period increased the specific yield by only 1 to

3 percent over that obtained from short term drainage. For fine materials, specific yield by saturation was about 3 times as great as that obtained by unwatering, apparently because the columns were too short and were primarily occupied by the capilliary fringe. Average values of specific yield obtained were: gravel and coarse sand, 34.8 percent; medium and fine sand, 24.2 percent; very fine sand, silt, and clay, 4.2 percent.

Laverett (1941) used pairs of tubes 10 feet long and of 0.75 inch diameter filled with sand. Sand in one tube of each pair was saturated with water and allowed to drain for at least 2 weeks whereas the initially dry sand in the other tube was fed water as far as the sand would imbibe it. His depth-moisture content curves are of the same general character as those of King (1899), but he found that the curves were not the same for the drainage conditions as they were for the imbibition conditions- a hysteresis zone was formed.

Drainage tests of unconsolidated Wilcox sand packed in pipe columns 8 feet long and 2 or 4 inches in diameter were made by Stahl, Martin, and Huntington (1943). The columns were equipped with water jackets to maintain temperature control. Holes were drilled and plugged at 6 inch intervals vertically so that cores of sand could be obtained at any time during drainage. The sand packed columns were saturated with fluid from the bottom, drained for several hours, and saturated again, prior to each drainage run. Data relating retained moisture to time of drainage were obtained by removing core samples after drainage for given intervals of time. The cavities caused by taking the cores were filled with the fresh sand before the columns were resaturated and drained for a new interval of time. The rate of drainage was found to be proportional to the temperature.

Coleman (1946) packed air-dry clay-loam soil in a brass tube 7 feet long and 6 inches in diameter. Holes were drilled in the tube at 6 inch vertical intervals to provide access for the installation of tensiometers and to obtain samples for moisture content. The column was saturated and drained under a variety of moisture tensions from 0 to 160 cm of water. On the basis of the plot of soil moisture versus pressure potential for the four soil columns at the end of the drainage periods being subjected to various drainage tension values, he concluded that the most appropriate value for discharge, tension seemed to be about 125 cm of water, which is equivalent to the pressure potential of -125 gm-cm/cm at the 19 percent (field capacity) moisture content. As shown by the author, the pressure potential at field capacity or moisture equivalent depends upon the moisture content representing these values, decreasing as the field capacity or the moisture equivalent increases. Thus a column filled with sandy soil would require a lower tension to drain than would one filled with a soil of finer texture. Gatewood et al. (1950) obtained three undisturbed columns of alluvium, each 42 inches in length and 14 inches in diameter. The columns were saturated from the bottom for 15 days, then allowed to drain, and the decline in water level was observed for about 25 days. Water levels in the 3 columns reached drainage equilibrium at the end of 9, 10, and 15 days respectively. The volume of water withdrawn from each column was divided by the volume of material unwatered and multiplied by 100 to obtain the coefficient of drainage, in percent, for the period of drainage.

Lamb (1951) drained 2.5 inch diameter saturated soil columns with piezometers spaced vertically on the side. He measured and recorded the quantity of outflow and the pressure heads at selected intervals in the column. Pressure heads below the visual line of saturation were measured by piezometers and it was pointed out that the visual line of saturation was not the true line of saturation because a considerable amount of drainage occurred after the visual line of saturation appeared to have reached its equilibrium position.

Terwillinger et al. (1951) packed a clean silica sand, by means of mechanical vibration, into a vertical plastic tube 13 feet long and 2 inches in diameter. To eliminate the possibility of boundary flow along the wall of the tubing, the plastic was heated and was forced under pressure to conform to the outside surface of the sand. The column was saturated with a 0.25 normal sodium chloride solution and then allowed to drain. When drainage equilibrium was reached, the saturation distribution was obtained by electrical conductivity readings for probes spaced at 5 cm intervals down the length of the column.

In 1952, Arnold Klute achieved the numerical solution to the flow equation for water in unsaturated media. The sand was assumed to have an initially constant moisture content of 0.01 gm/gm. The end at $X = 0$ was considered to be maintained at saturation, $\theta = 0.357$ gm/gm. The conductivity moisture content function calculated according to Child's method and the specific capacity moisture content function from a desorption curve were used to calculate $D(\theta)$.

By the use of tensiometers, Luthin and Miller (1953) measured the volume of outflow and recorded the pressure distribution in soil columns, about 8 cm in diameter and 120 cm in length, during drainage. Null-type tensiometers were installed at intervals in the columns to determine moisture distribution during drainage of the column. As drainage proceeded the upper part of the soil column became unsaturated, and the water drained until the capillary forces resisting the downward movement of water were sufficient to neutralize the downward forces. The hydraulic gradient approached zero in the lower part of the soil column and the hydraulic conductivity remained high. Water continued to move out of the upper zone under the driving action of a high hydraulic gradient, but movement was slow because of a reduced hydraulic conductivity. Static equilibrium was reached when the hydraulic gradient was zero throughout the column. At this time the soil column was separated into sections, the moisture content was determined, and the percentage of pore saturation was calculated.

Day and Luthin (1956) did their experimental drainage work with a column of Oslo Flaco sand, about 87 cm high and 7 cm in diameter. Tensiometers were installed at height of about 5, 20, 36, 51, 65, and 83 cms from the bottom of the column. The column was saturated, and then drained while the tension and outflow volume measurements were made at selected times. The data were then used to test a numerical solution of a differential equation for vertical flow.

Marx (1956) used a continuous sandstone cylinder, 154 cm long and 5 cm in diameter, with its lateral surfaces sealed in plastic tubing. The volume of water draining from the saturated core was obtained by weighing. The data obtained from the column drainage were compared with drainage data obtained from centrifuging small samples of the same sandstone. The correlation between centrifuge prediction and actual gravity drainage, on the basis of the development presented in the paper, may be considered to be neither more or less precise than the sampling technique itself. To each small, reconstituted sample, for any given centrifuge accelaration, there will correspond an ideal, gravity drained prototype column, and the centrifuge drainage of the samples can be used to predict the gravity drainage of this ideal prototype, provided the boundary conditions are satisfied.

Jacob Bear (1972) concluded that, for flow through the porous media, the assumption of uniform saturation along the column was a good approximation only for sufficiently high values of flow rates for nonwetting and wetting fluids. The assumptions failed for low specific discharges. This phenomenon has been defined as capillary end effects. When a wetting fluid displaces a nonwetting one from the column initially saturated by the latter, the wetting fluid will not flow out until S_w at the outflow face i.e., $X = L$ has been built up to some critical value. S_w is thw wetting fluid saturation. Initially only the nonwetting fluid is present at $X = L$ and therefore we have $P_c = 0$. P_c is the capillary pressure. Cosequently, no wetting fluid will appear outside the outflow face until $p_c = 0$ is also reduced to zero just inside the outflow face. As S_w increases during the experiment, the imbibition curve is applicable. However, at this critical value of nonwetting fluid saturation; the nonwetting fluid forms a discontinuous phase and ceases to flow. From this discussion, it follows that as long as $S_w < (1-S_{nwo})$ at the outflow face, only nonwetting fluid can leave the column through its outflow face. S_{nwo} is the residual saturation of non-wetting fluid. However, at $S_{nw} = S_{nwo}$, the permeability k_{rnw} of the nonwetting fluid becomes zero. This means that an infinite pressure gradient must exist in the nonwetting fluid if this fluid is to flow at a saturation of S_{nwo}. The same anology can be applied to the flow of wetting fluid through a column of porous media.

In 1960, Liakopoulos derived the flow equations for the movement of water through anisotropic unsaturated soils. The governing equations were of such a nature that a solution exists for time, $t > 0$, and is uniquely determined if two relationships are defined together with the specified state of the system at the initial time, $t = 0$, and at the boundaries. The two required relations are of pressure versus hydraulic conductivity and pressure versus volumetric water content. However, he recommended that, because of strong non-linearity in its terms, the equation be solved for various initial and boundary conditions by approximating the differentials with finite differences at discrete points in the solution domain. Either a relaxation or an iteration procedure can be used to get the solutions.

Johnson, Prill, and Morris (1963), made a preliminary study of column drainage in which factors such as cleaning of media, methods of drainage, column diameter, method of wetting, method of packing and length of drainage periods were evaluated. They found that (1) cleaning with acid slightly affected the drainage characteristics for glass beads; (2) the diameter of the column made little difference in the moisture distribution after drainage of mechanically packed 1, 4, and 8 inchs diameter columns of 0.120 mm glass beads; (3) different procedures for wetting the porous media gave similar results for the distribution of water after drainage and for the rate of discharge; and (4) a mechanical method of packing produced repeatability of moisture contents.

Scott and Corey (1960) investigated the distribution of pressure during steady flow in unsaturated sands. They derived the differential equation for steady flow in porous material occupied by two immiscible fluids such as air and water. However, they assumed that Darcy's equation applies simultaniously to the wetting and nonwetting phase. Experiments were conducted using a hydrocarbon liquid as the wetting fluid, air as the nonwetting fluid in a 5 ft long column of uniformly graded sand and in a system of coarser sand overlain by a finer sand porous media. They found that the curves for relative permeability versus capillary pressure were linear for the sands investigated. Also, they concluded that when the steady flow is downward through a column of unsaturated sand, the effective permeability tends to reach the same value in each stratum provided the strata are sufficiently thick. At the bottom of a coarse textured stratum underlain by a finer textured stratum, however, there will develop a region of low saturation and low permeability. The reason for this is that the relationship between capillary pressure and depth (Z) must be continuous regardless of abrupt changes in texture.

Rose (1962) questioned the classical description of fluid displacement processes. He derived the relevent displacement equation from mass conservation and continuity statements. Darcy's law for the description of unsaturated flow was suspected on theoretical grounds, both in its analytical form, and in the way it is commonly used to describe the steady unsaturated displacement processes. The two main aspects of the research were the linearity function used to describe the saturation dependent relative permeability (K_i) and the mobility ratio parameter (M). He described the basic difficulties during instrumentation to obtain the above two functions rendering the classical equation determinate.

From those investigators who were not prepared to accept this conclusion, he demanded laboratory data that conclusively showed that the functions are independent of the viscosity ratio and the degree of interfacial contact between contiguous immiscible fluids, independent of the degree of collinerity between the flux and force vectors of the contiguous fluids (giving rise in extreme cases to cross flow and counter flow displacements) and independent of the magnitude of the local saturation changes during displacement.

None of these displacement factors are even embodied in the sense of Darcy's law. The last aspect deals with the meaningfulness of the saturation dependent capillary pressure function. He concluded that the classical theory is based on mixed incompatible parts, which expose the inherent ambiguities. He also concluded that, other, more general laws of force are needed in place of Darcy's law to be combined with the mass conservation statement.

In 1962, Hanks and Bowers obtained the numerical solution of the moisture flow equation for infiltration into layered soils. Solutions were obtained for infiltration into a loam over a silt loam and for silt loam over loam. These showed that the infiltration was governed by flow through the less permeable soil, provided the wetting front had extended well into the second layer. The results were excellent when compared to the results obtained by Scott et al. (1962) and Philip (1955) for horizontal infiltration into homogeneous soils at a uniform initial water content.

In 1963, Elrick conducted experiments to determine the unsaturated flow properties based on the steady state Darcy's equation or on the transient diffusion equation. He concluded that results based on transient outflow pattern at high moisture contents (using a coarse sand) differed considerably from that predicted by diffusion theory. Also the extended theory, which takes into account the air as well as the water phase, does not agree with the observed transient behavior at high moisture contents. However, at low moisture contents the agreement between the two methods is considerably better.

Although water tables as zero-pressure surfaces are traditionally taken as boundary conditions in the analysis of open flow systems, they do not actually separate between saturated hydraulic conductivity (K_s) and impermeability. Bower (1964) concluded that in negative -pressure regions above the water table, a functional relationship exists between the hydraulic conductivity (K) and the soil-water pressure head (P). The relationship between K and P is a characteristic of each individual soil and must, in principle, be experimentally determined. Experimentally determined relationship between K_p and P can be included in the solution of flow systems by iterative procedures, which can be carried out with a digital computer or a resistance network analog. K_p was defined as the conductivity of soil at negative soil-water pressure. Liquid flow in the negative pressure regions was, however, considered to be Darcian.

In 1965, Whistler and Klute derived a numerical solution to the flow equation for a flow system consisting of a vertical column of soil which had been drained from saturation to equilibrium with a water table. A thin layer of ponded water was assumed to be applied to the top of the column. The solution of the equation depicted the time and depth distribution of water content and pressure head during the resulting infiltration. The results showed that for soils that have water content-pressure head curves with regions of steep slopes, the wetting front advances as a steep, well defined front. In those soils with more gently sloping water content-pressure head curves, the wetting front was more diffused. If hysteresis is ignored, the position of the front at any one time is overestimated or underestimated, depending upon whether drainage or wetting curves are used.

Irvine Remson et al. (1967) published a numerical model of soil-moisture systems for the following conditions: (a) the model may be used for one-, two-, or three- dimensional unsteady flow systems, (b) the soil may be non-homogeneous, (c) sinks and sources may occur anywhere in the system, (d) irregular distribution of initial and boundary moisture content may be specified, (e) the influence of gravitational and capillary potentials is included, (f) non-linear and multidimensional relationships between matrix moisture content, capillary potential, and the hydraulic conductivity may be specified.

Rubin (1968) solved the Darcian flow equation for the two dimensional, transient transfer of water in rectangular, unsaturated or partly unsaturated soil slabs, numerically with the aid of an alternating direction, implicit difference method. Two alternative processes were considered: predominantly horizontal infiltration and ditch-drainage. The results obtained for the horizontal infiltration process indicated that it involved upward flow components which were due, primarily, to a gravity induced variation in hydraulic conductivity along the inflow face. These components may materially affect the course of infiltration, especially during its earlier stages. He concluded that the drainage case results demonstrated that transient water flow within the unsaturated zone and the outflow from the seepage zone may significantly affect the progress of water table descent and the total outflow rates.

Allan Freeze (1969) presented a one dimensional, numerical mathematical model involving transient flow through an integrated saturated-unsaturated system. He observed that water table fluctuations result when the rate of ground water recharge or discharge is not matched by the unsaturated flow rate created by infiltration or evaporation. A water table rise provides the source of replenishment to the groundwater zone that allows the prevailing rate of recharge to continue. This dynamic water table behavior was stimulated by the model and the solution were applicable to homogeneous, isotropic soils in which the functional relationship showed hysteresis properties. The model allowed upper boundary conditions of constant rate rainfall, ponded water, evaporation and redistribution. Also, the model can be used to determine the water table fluctuations that will arise from a given set of initial conditions, and soil type.

Blair (1969) solved the two-phase flow problems using an implicit difference equation. He made use of a Newtonian iterative technique for solving the resulting nonlinear set of algebraic equations that arise at each time step. He concluded that implicit difference equation in calculation of two-phase flow is entirely practical and the diffence equation is stable where a mixed equation is not.

Hornberger and Remson (1970) proposed a moving boundary model for a one-dimensional transient flow of water through a porous medium of which part is saturated and part is unsaturated. The model is based upon a theory that implies a discontinuous propogation of pore pressure at the saturated-unsaturated interface. Two numerical procedures were developed to solve the problem, an approximate Taylor series method and a finite difference method. The results were compared to the experimental data given by Watson (1967).

A finite difference model was derived by Green et al. (1967) to describe the isothermal, two-phase flow in porous media in the absence of evaporation and transpiration. The model solution was obtained with the aid of a digital computer using an iterative implicit procedure. A comparison was made between computed results and experimental field data on moisture movement beneath a shallow, surface pond. Water was added to the pond at controlled rates to maintain an approximately constant head for a set time period. Following this wetting period, the pond was kept dry but covered to reduce evaporation. At different times during the wetting and drying periods, neutron logs were run to measure water saturation versus depth at depths up to 22 feet. The experiment was simulated with the computer model and excellent agreement between calculated results and the data were obtained.

Jeppson (1970) emphasised the hydraulic conductivity and capillary pressure relationship for analyzing moisture movement through unsaturated soils. He used the Burdines integral to the obtain hydraulic conductivity and capillary pressure relationship from saturation-capillary pressure relationships. Jeppson developed a computer program to evaluate the Burdines integral using discrete data which define the saturation capillary pressure relationship for a given soil.

An algorithm for the prediction of infiltration and water content profiles has been derived from the two-phase flow theory of water and air by Morel-Seytoux, and Billica (1985). However, the governing equation can be reduced to the usual one phase flow equation by setting a parameter to a zero value. The need for the two-phase approach is demonstrated for a couple of illustrative situations with a semi-infinite lower boundary condition even when air cannot be confined. Rather amazingly, the two-phase computer run cost less than the corresponding one-phase run. The two-phase approach is less expensive and more realistic.

CHAPTER III

MATERIALS AND METHODS

The objective of the research is to test the hypothesis that, for any point above a height equivalent to the vapour pressure head of water above the water table, Darcy's law is not valid for unsaturated soil media. Movement of water in the unsaturated zone of soil media is mainly due to capillary phenomenon and the values of capillary pressure or matric potential are negative. In the region above the water table, the continuity of water and thus the capillary pressure is maintained as long as the pressure values are more than the vapour pressure of water. As the capillary pressure falls below the vapour pressure, the continuity of water as a liquid is disrupted and the phenomenon of vapour movement takes place. The discontinuity of water implies that the differentials of pressure (i.e., hydraulic gradient) cannot exist under such circumstances, thus the validity of Darcy's law for such regions is questionable. Movements due to temperature gradients have been minimized by placing the column of soil in a constant temperature environment.

Experimental Design

The experimental system was designed to demonstrate that for long columns the continuity of water or the hydraulic gradient for the top region is questionable. Further, the validity of Darcy's law has to be ascertained for unsaturated soil conditions. A 30-ft high column has been assembled using 1-ft long and 9-inch outer diameter and 8-inch inner diameter steel cylinders as shown in the Figure 3.1. The column length of 30-ft was chosen because of the fact that it exceeds the net positive suction head (i.e., 29.67 ft head of water) for Lubbock, Texas. Net positive suction head is the difference between the normal atmospheric pressure and the standard vapour pressure. Normal atmospheric pressure can be expressed as a 30.26 ft head of water at an elevation of 3254 ft above MSL for Lubbock, Texas. The standard vapour pressure is taken as 0.59 ft head of water.

The column is situated in a 45-ft deep, 5-ft diameter cylinder which is capped at the bottom. A constant temperature is achieved by locating the column in such a place. Additionally every alternate section has been equipped with a temperature sensing device which may be read using a digital thermometer.

During the process of assembling the sections, extra care was taken to make the joints across the flanges watertight. This has been achieved by using silicon silex

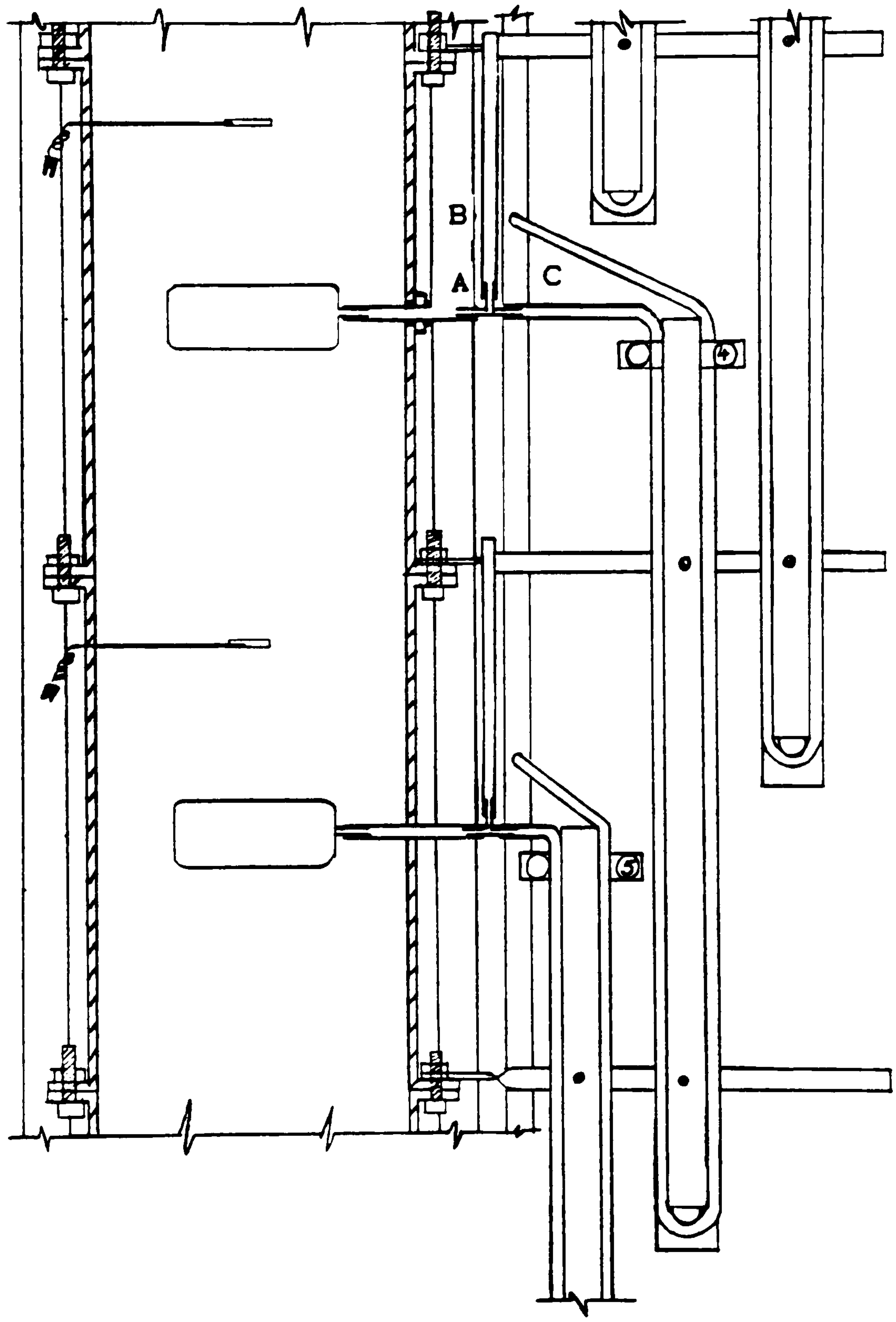


Figure 3.1 Section of Sand Column

adhesive. The sections were filled with sand and vibrated using an external vibrator to achieve approximately constant values for porosity.

Each section has a porous cup embedded in the sand and connected to a manometer for measuring pressure potential. Thus the column has 30 mercury manometers attached to the column to measure the variation in pressure potential. Each manometer consists of three transparent plastic tubes (see Figure 3.1). Tube A is connected to the porous cup and has a plastic tee attached to the other end. Tube C contains mercury and has two limbs which run along the edges of a wooden scale to facilitate measurement of pressure head. One end of the tube is attached to the tee joint and the other end is attached to a PVC pipe initially open to the atmosphere at the top and which serves to collect any spillage of mercury. Tube B is held in a vertical position for bleeding air from the manometer. Once the column was saturated and under equilibrium conditions, manometer tubes were deprived of entrapped air; the tubes were filled with water, and tube B was sealed air tight at the open end. The column has two aluminium tubes attached outside it which were used to carry the gamma probe and sensor. The gamma probe (Troxler, Model-2376) provides the density measurements. The density of the sand at different locations was used to compute the moisture content of the sand media with respect to time and location. The temperature inside the well remained relatively constant throughout the year.

Soil

Wind-blown sand was selected for this study because it is readily available in this region. It was obtained from sand dunes near Brownfield, Texas. The particle size distribution of the sand varies from 75 microns to 4.75 mm. The sand is light brown in color and the particle size distribution curve is shown in the Figure 3.2.

Water Application

Hoses 1-inch in diameter were attached to the top cap and the bottom of the steel column for saturating and draining the column. A plastic collector bucket was attached to a hose connected to the bottom of the column. The bucket could be moved up or down the height of the column for controlling the discharge and seepage rates. The elevation of the bucket was recorded for the drainage experiments. Water from Lubbock's municipal supply was used for experimentation purposes.

Measurements

The process of data collection was performed in two steps. In the first step, the column was compacted to attain approximately the same dry density. Dry density

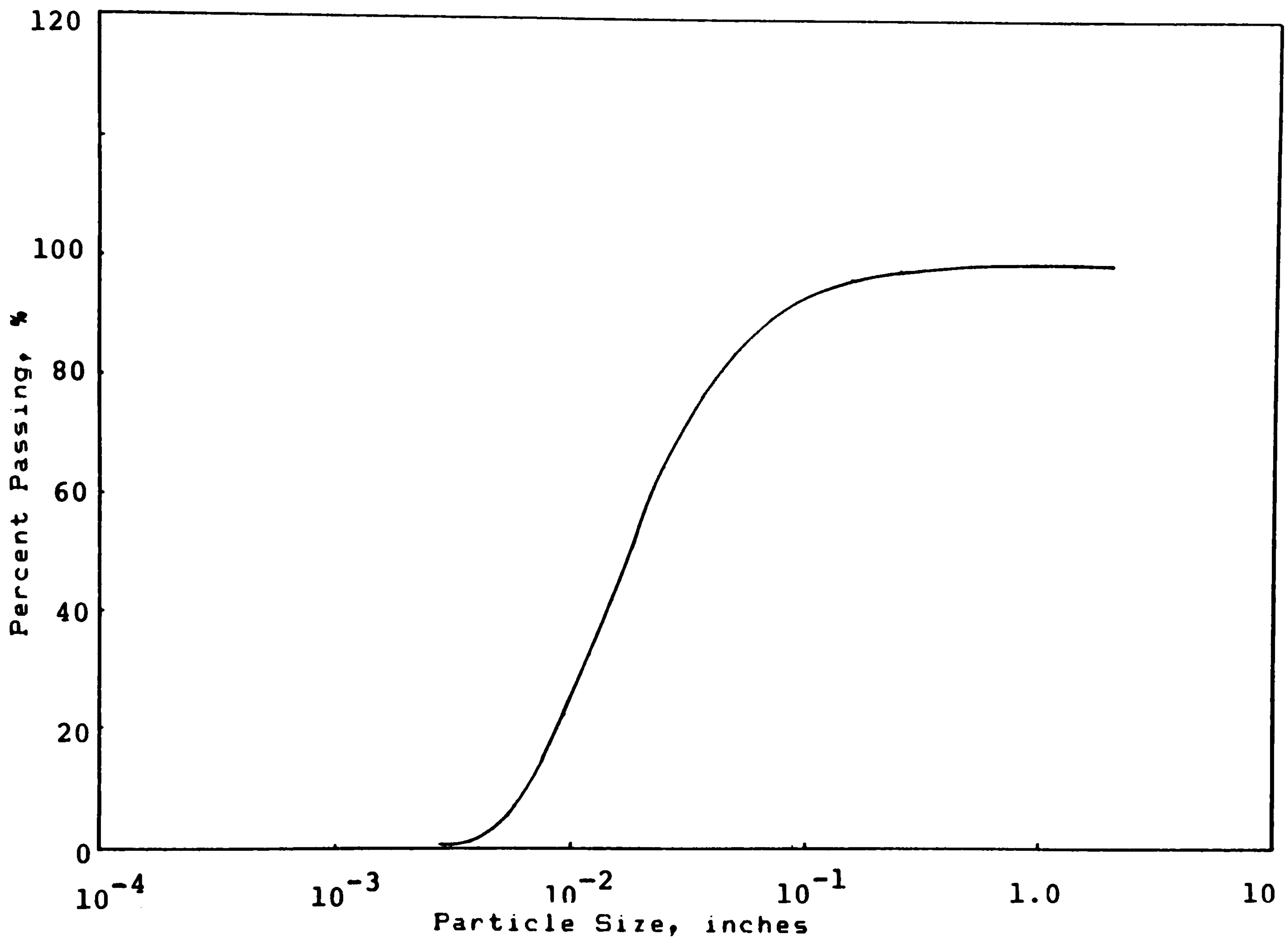


Figure 3.2 Particle Size Distribution

readings were obtained using the gamma probe. In the next step, the column was saturated from the bottom and a density log of the column was obtained. The difference betweem the two density readings at measurement points along the column (saturated and dry) gave the values of porosity which were found to vary along the height of the column. The two steps in the data collection process are discussed in the following paragraphs.

Step I

Measurements of changes in head of water with time were made for saturated, steady-flow conditions to obtain the value of saturated conductivity. The process basically follows the principle of the falling head permeameter (Bear, 1972). A plot of log(ΔH) versus time was plotted and slopes of the resulting curves were compared for upward and downward flow conditions.

Step II

Drainage tests were conducted begining with the saturated column. Density and discharge readings were taken daily. The amount of water drained from the bottom of the column was measured volumetrically. A Troxler type two-probe density gauge, Model-2376 was used to measure the soil-water content on volume basis. The probe readings were taken at 1-ft intervals. Manometer readings were obtained at the same time as the gamma probe readings.

The drainage test was repeated several times until the final set of readings were collected. Treatment of the sand column was carried out during the second year of the column's existence after excessive biological growth was noticed in the manometers. A 0.01N calcium hypochlorite solution was used as disinfectant and allowed to drain through the column. Drainage experiments were resumed after disinfection.

CHAPTER IV

MATHEMATICAL MODEL DEVELOPMENT AND COMPUTER PROGRAM

The mathematical model derived in this study is based on the equation of continuity and Darcy's equation, which are assumed valid for both saturated and unsaturated flow conditions. Application of Darcy's equation for unsaturated soil media in the test was based on the following simplifing assumptions:

1) The fluid medium is continuous,
2) The soil matrix is rigid and continuous,
3) The fluid is in motion,
4) The fluid is incompressible,

For studies in the unsaturated zone, soil scientists have successfully used a capillary-or matric-potential based formulation. The usual procedure in a potential based formulation is to use matric potential as the independent variable.

Model Development

The equation of continuity is a statement of the principle of conservation of mass, Figure 4.1. For a saturated porous medium, the equation is written as

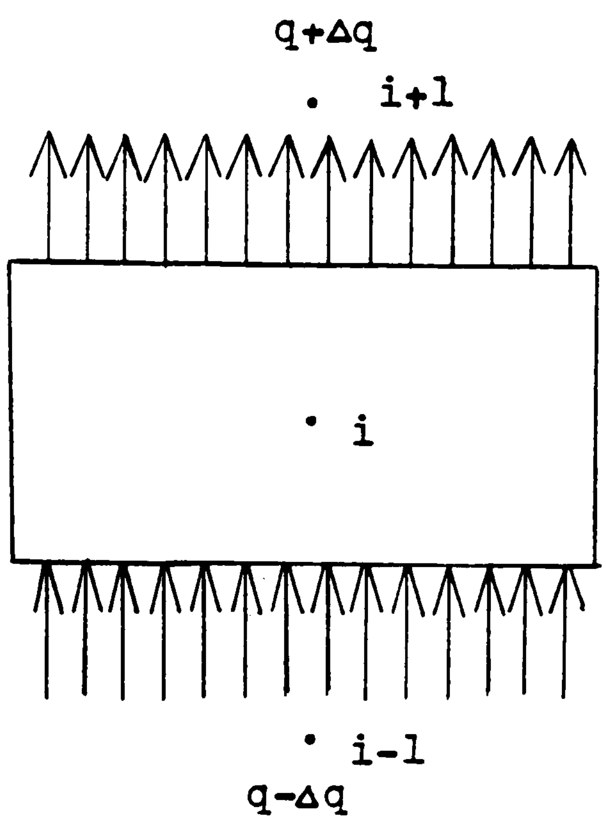


Figure 4.1 Conservation of Mass

$$-\frac{\partial q}{\partial Z} = \frac{\partial \theta}{\partial t} \tag{4.1}$$

where θ is the water content on a volume basis, and q is the volume flux of fluid flowing through a unit cross section normal to the direction of flow in unit time. Darcy's equation for flow of fluid in unsaturated porous media can been expressed as follows (Hillel, 1982):

$$q = - K(h)\frac{dH}{dZ} \quad . \tag{4.2}$$

The terms on the right hand side of the equation are the total potential gradient (dH/dZ) and the coefficient of conductivity (K(h)). Total potential (H) is expressed as the sum of the pressure potential (h) and gravitational potential (Z). When soil water is at a hydrostatic pressure level greater than atmospheric pressure, its pressure potential is considered to have a positive value and is called the submergence potential (Hillel, 1982). Negative pressure potential is called the capillary or matric potential. Equation (4.1) and (4.2) can be combined to yield the equation of flow for fluid in an unsaturated porous media:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left(K(h)\frac{dH}{dZ}\right) \quad . \tag{4.3}$$

Substituting H = h + Z, the equation becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial K(h)}{\partial Z}\left(\frac{\partial h}{\partial Z}+1\right) + K(h)\frac{\partial^2 h}{\partial Z^2} . \qquad (4.4)$$

If θ and K can each be considered to be related by single valued functions to h, then Equation (4.4) can be written in the form

$$\frac{\partial h}{\partial t} = \frac{dh}{d\theta}\left(\frac{dK(h)}{dh}\ \frac{\partial h}{\partial Z}\left(\frac{\partial h}{\partial Z}+1\right) + K(h)\frac{\partial^2 h}{\partial Z^2}\right) \qquad (4.5)$$

This non-linear partial differential equation represents the vertical, one-dimensional, isothermal movement of water in a rigid, homogeneous unsaturated porous system. The vertical ordinate Z has been taken as positive upwards. The coefficient of conductivity (K(h)) is a function of fluid viscosity, density, intrinsic permeability, and capillary or matric potential.

Substituting B1, B2, and B3 for the $\frac{dK(h)}{dh}$, K(h), and $\frac{d\theta}{dh}$ respectively, Equation (4.5) can be written for coordinate (i,j+1/2)

$$\left(\frac{\partial h}{\partial t}\right)_{i,j+1/2} = \frac{1}{B3_{i,j+1/2}}\left(B1_{i,j+1/2}\left(\frac{\partial h}{\partial Z}\right)_{i,j+1/2}\right.$$

$$\left(\left(\frac{\partial h}{\partial Z}\right)_{i,j+1/2}+1\right) + B2_{i,j+1/2}\left(\frac{\partial^2 h}{\partial Z^2}\right)_{i,j+1/2}) \qquad (4.6)$$

where i and j are the spatial and time coordinates.

Using the Crank-Nicholson method to represent equation (4.6), the finite difference form for $\left(\frac{\partial h}{\partial Z}\right)_{i,j+1/2}$, $\left(\frac{\partial h}{\partial t}\right)_{i,j+1/2}$, and $\left(\frac{\partial^2 h}{\partial Z^2}\right)_{i,j+1/2}$ can be written as (Carnahan et al., 1969):

$$\left(\frac{\partial h}{\partial Z}\right)_{i,j+1/2} = \frac{1}{2DZ}((w(h_{i+1,j}-h_{i-1,j})+$$

$$(1-w)(h_{i+1,j+1}-h_{i-1,j+1}))$$

$$\left(\frac{\partial h}{\partial t}\right)_{i,j+1/2} = \frac{1}{Dt}(h_{i,j+1}-h_{i,j})$$

and

$$\left(\frac{\partial^2 h}{\partial Z^2}\right)_{i,j+1/2} = \frac{1}{DZ^2}(w(h_{i-1,j+1}-2h_{i,j+1}+h_{i+1,j+1})+$$

$$(1-w)(h_{i-1,j}-2h_{i,j}+h_{i+1,j})) .$$

The constants can be expressed as:

$$B1_{i,j+1/2} = \frac{1}{2}(B1_{i,j}+B1_{i,j+1})$$

$$B2_{i,j+1/2} = \frac{1}{2}(B2_{i,j}+B2_{i,j+1})$$

and

$$B3_{i,j+1/2} = \frac{1}{2}(B3_{i,j}+B3_{i,j+1}) \quad .$$

DZ is the element height or the distance between two nodes and Dt is the time step. Using the finite difference forms for the differentials, Equation (4.6) can be written in the general form as:

$$A(I)h_{i-1,j+1}+B(I)h_{i,j+1}+C(I)h_{i+1,j+1} = E(I) \qquad (4.7)$$

where,

$$A(I) = \frac{-B1}{4DZ^2B3}(-2wh_{i+1,j}+2w^2h_{i+1,j}+2wh_{i-1,j}-$$

$$2w^2h_{i-1,j}+h_{i-1,j+1}-2wh_{i-1,j+1}+w^2h_{i-1,j+1}$$

$$-2DZ+2wDZ+(-1+2w-w^2)h_{i+1,j+1}+\frac{4B2w}{B1})$$

$$B(I) = (\frac{1}{DT}+\frac{8B2w}{4DZ^2B3})$$

$$C(I) = \frac{-B1}{4DZ^2B3}(2wh_{i+1,1}-2w^2h_{i+1,j}-2wh_{i-1,j}+$$

$$2w^2h_{i-1,1}+h_{i+1,j+1}-2wh_{i+1,j+1}+w^2h_{i+1,j+1}$$

$$+2DZ-2wDZ+(-1+2w+-w^2)h_{i-1,j+1}+\frac{4B2w}{B1})$$

$$E(I) = \frac{B1}{B3}(-w^2h_{i+1,j}+w^2h_{i-1,j}-2wDZ-2wDZ$$

$$+\frac{4B2(1-w)}{B1})h_{i-1,j} + (\frac{4B2(2w-2)}{B3}+$$

$$\frac{4DZ^2}{Dt})h_{i,j} + (\frac{B1}{B3}(wh_{i-1,j} - w^2h_{i-1,j}$$

$$+2wDZ+\frac{4B2(1-w)}{B1})h_{i+1,j} \quad .$$

The values of w vary from 0 to 1 and control the discretization error thus facilitating the solution technique. The differential (Equation 4.7) when applied to a vertical column of height L, results in a tridiagonal-banded coefficient matrix which can be solved by the implicit, Gauss-elimination method.

The program handles the nonlinearity of the partial differential equation by assuming the values of $h_{i-1,j+1}$, $h_{i,j+1}$, and $h_{i+1,j+1}$ equal to the respective values of $h_{i-1,j}$, $h_{i,j}$, and $h_{i+1,j}$ to compute the values of the coefficients A(I), B(I), and C(I) for the first iteration. The program solves the resulting tridiagonal matrix for new values of $h_{i-1,j+1}$, $h_{i,j+1}$, and $h_{i+1,j+1}$ and thereafter the new values are improved upon in successive iterations. The same procedure is repeated for each time step.

The initial and boundary conditions for the experimental setup can be defined as:

(a) Initial conditions for t = 0

$$h_i = \gamma(L-Z), \quad i = 1 \text{ to Node}$$

where L is the height of the column,

Node is the total number of nodes,

γ is the specific weight of water.

(b) Boundary conditions for t $\neq$ 0

$$\left(\frac{dH}{dZ}\right)_{Node} = 0 \quad \text{and}$$

$h_1(t)$ = a controlled step function value.

In order to implement boundary conditions for times greater than zero, the pressure potential value at node 1 (i.e., at the bottom of column) was assumed equal to the height of the collector bucket. It was also assumed that the change in total potential with respect to gravitational potential (Z) is zero at the top of the column. The program handles the equation of no-flow at the top by incorporating an imaginary node above the top of the column and making the total potential values for the top two nodes equal at each time step.

Computer Program

An implicit iterative technique has been utilized for the solution of Equation 4.7. The computer code is written in BASIC and the flow chart for the program is shown in Figure 4.2.

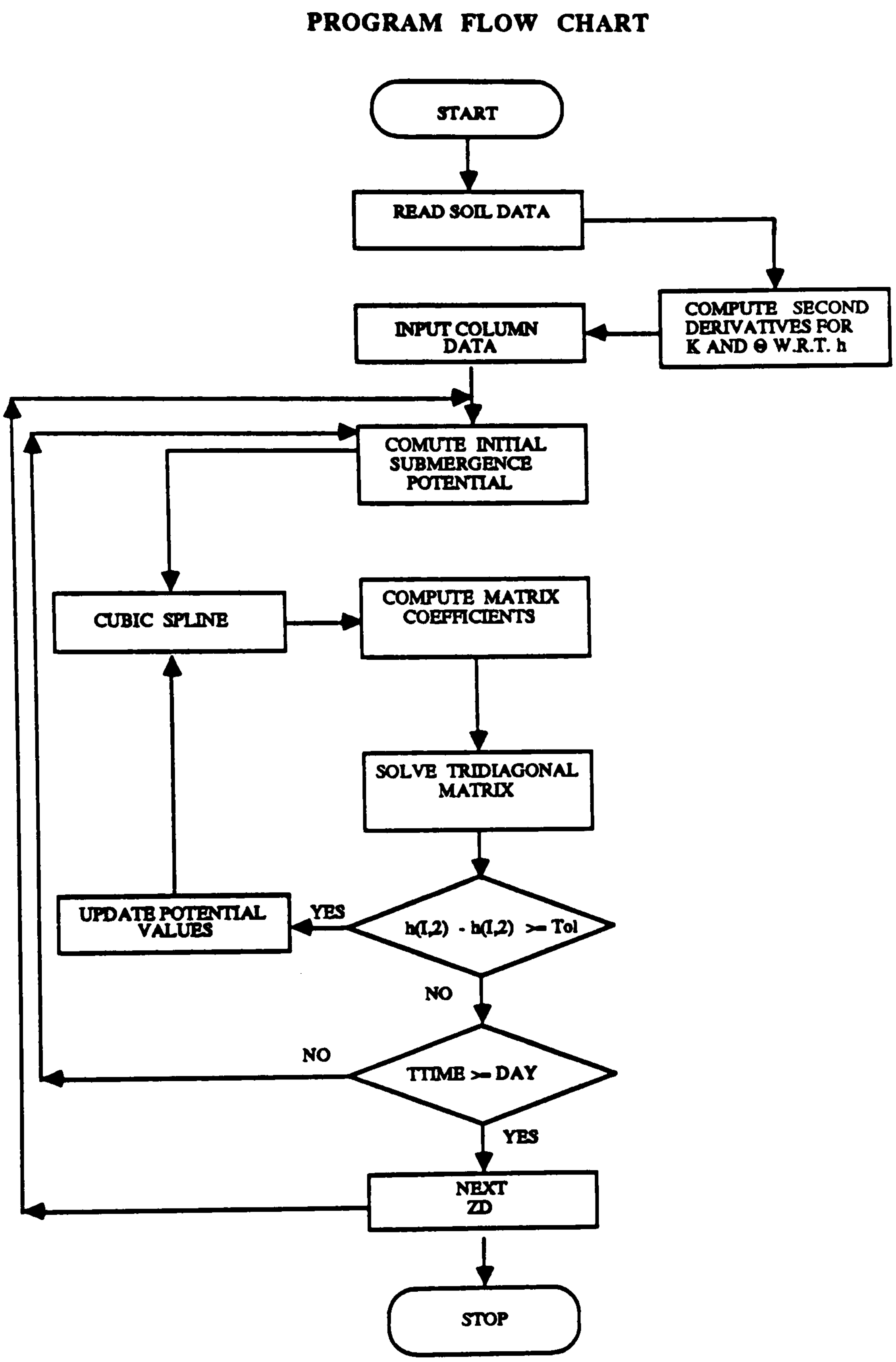


Figure 4.2 Flow Chart

Components Of The Program

Input Data:

Input data consists of the following parameters.

L = length of the column (ft),

Node = number of nodes,

N = number of data points for K versus h and θ versus h relationship,

Dt = time step (sec),

w = convergence parameter (dimensionless),

alpha = acceleration parameter (dimensionless),

The data for K versus h and θ versus h was taken from Watson (1967) for Botany sand which is similar to the one used in the experimental unit. The soil characteristic curves were extrapolated to a matric potential head of 30 feet and of 0.05 feet on each end of the curves (see Figure 4.3 and Figure 4.4). The actual matric potential head values ranged from 3.7182 to 0.9625 feet. The additional data points were added and the data was smoothed to facilitate the computation of second derivatives.

Cubic Spline Interpolation Subroutine

This subroutine computes the value of K(h) or θ for any value of h computed by the main program. The subroutine uses a 3rd degree polynomial to interpolate between each pair of data points. The second derivative within each

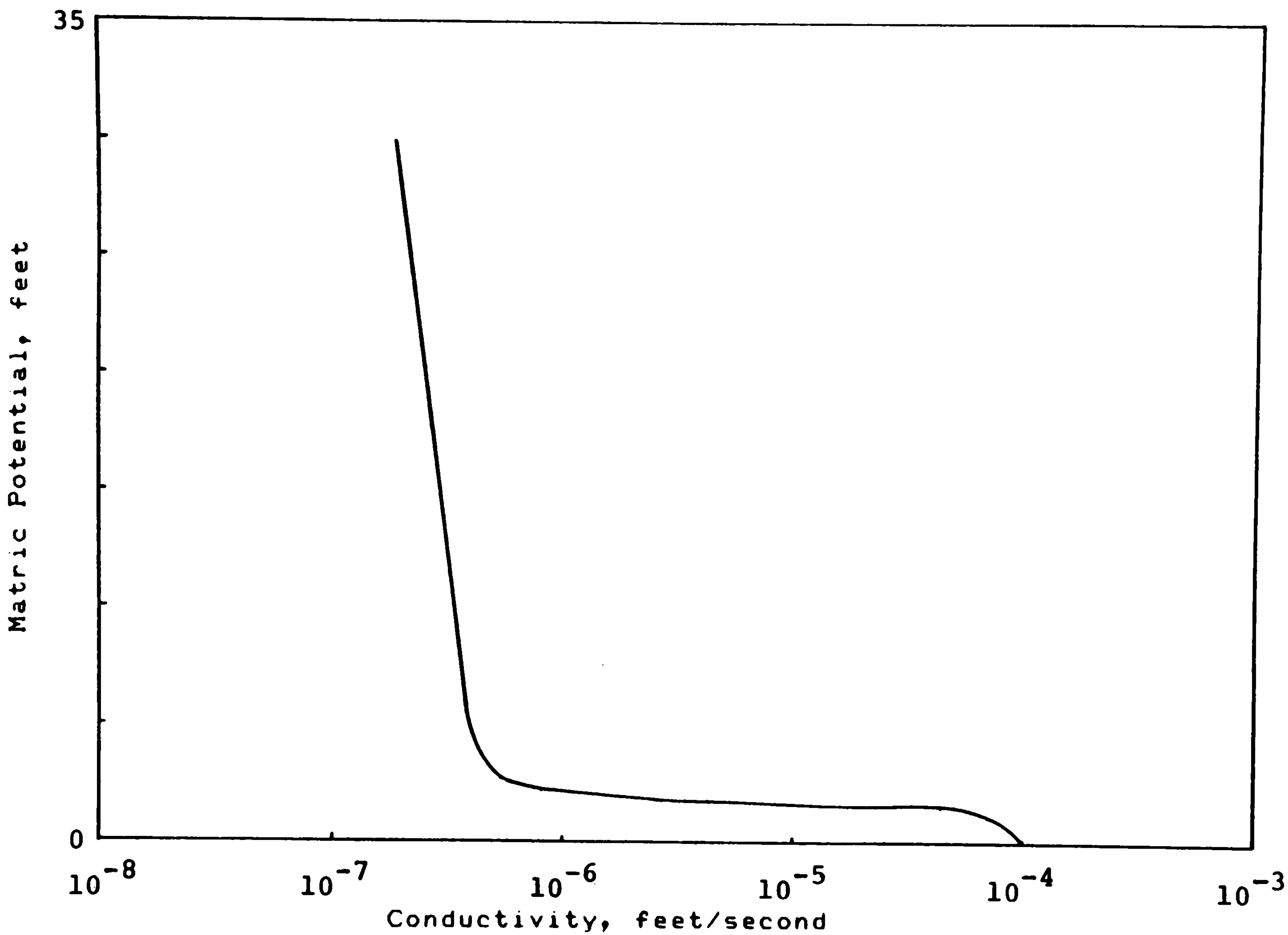


Figure 4.3 Matric Potential versus Conductivity

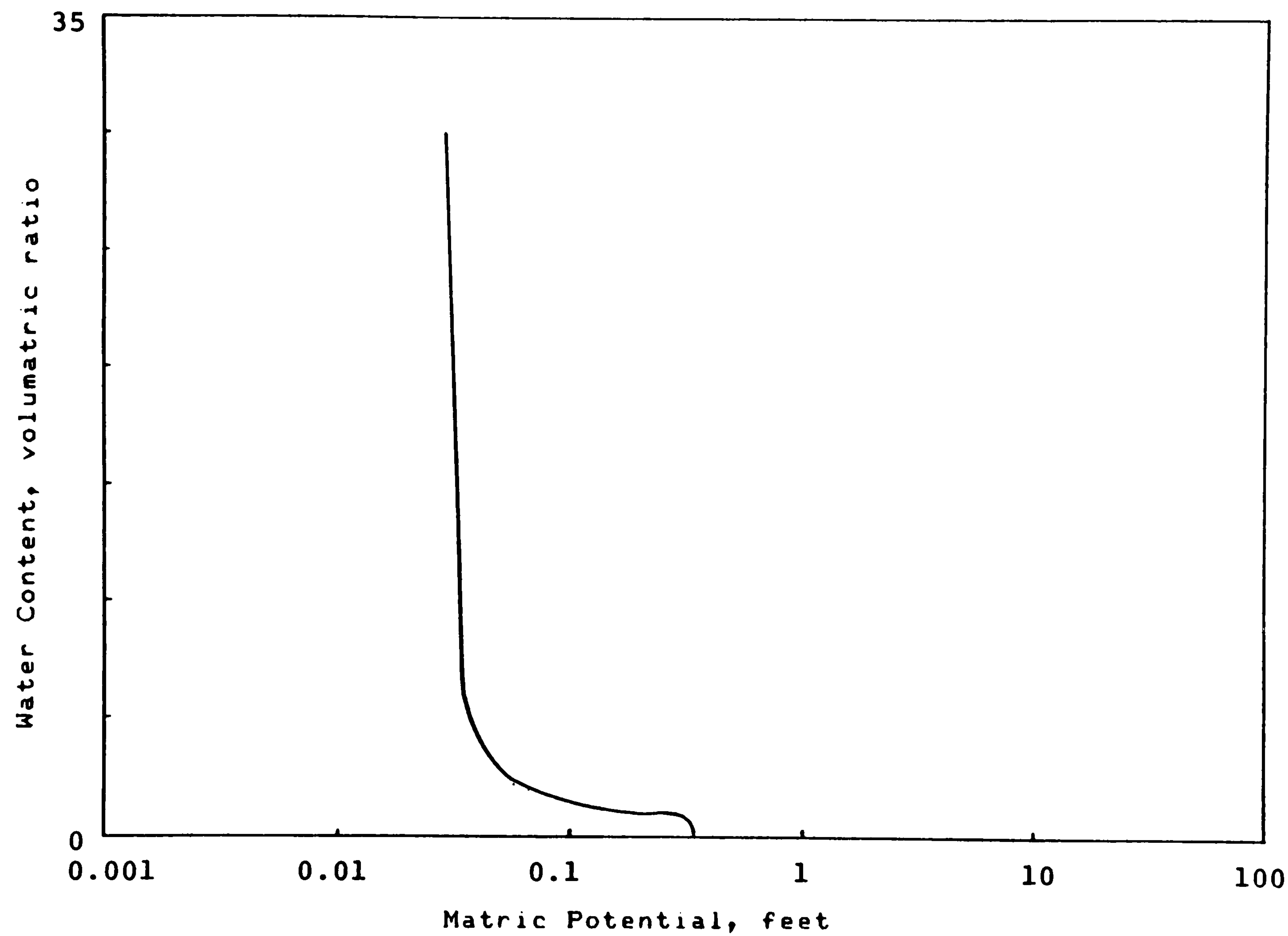


Figure 4.4 Matric Potential versus Water Content

interval is a constant. The slopes between two adjacent points can be obtained by differentiating the polynomial (Chapra et al., 1976).

Solution of Tridiagonal Matrix Subroutine

A banded matrix is a square matrix that has all its elements equal to zero, with the exception of a band centered on the main diagonal. In this case, the band width is three, thus the name Tridiagonal matrix. These matrix systems are frequently encountered in engineering and scientific practice, more often for finite difference solutions of partial differential equations. The tridiagonal matrix is solved by Gauss elimination method. However, due to the systems unique structure, the algorithm for implementing Gauss elimination can be simplified greatly, and solutions are obtained in a very efficient manner. For such a system, the forward elimination steps are simplified because most of the matrix elements are already zero. Then the remaining unknowns are evaluated by backward substituion (Carnahan et al., 1976).

CHAPTER V

RESULTS

The experimental system and the mathematical model described in Chapter III and Chapter IV have been used to test the validity of Darcy's law for flow in unsaturated media. The results of this analysis are described in this chapter.

Experimental Results

Bulk density readings have been collected over the period of drainage using a Troxler type two-probe density gauge Model-2376. Daily counts of bulk density readings were made at 59 nodes distributed along the height of the soil column at a node spacing of 0.5 ft. Subtracting the dry density readings from the daily bulk density value for a particular node yields the water content in lb/ft^3 for each day. These water content readings have been computed for about 70 days of drainage until the discharge from the column was low (about 40 ml on the last day). The amount of water draining from the column was volumetrically measured each day. Pressure data collected from the manometer was unreliable because of the inability to bleed air from the tubes.

The presence of air causes a discontinuity of water in the manometer tubes thus giving unreliable potential head values. Thus only density readings were collected. The experimental data collected in the study consists of the following:

1) Time variation of collector bucket elevation, (Figure 5.1),
2) Time variation of discharge, q, based on actual and on gamma probe measurements (Figure 5.2),
3) Cumulative discharge, Q, versus time based on the actual and on gamma probe measurements (Figure 5.3), and
4) Time variation of water content for each node (Figure 5.4 to Figure 5.13).

The following observations have been made from an analysis of the experimental data.

1) The water content in pounds per cubic feet is not constant based on the bulk density count for the saturated column. Porosity was found to vary from 0.20 at the top to 0.385 at the bottom of the column. One reason for this could be the excessive biological growth that occurred during the second year of the column's existence. Anarobic decomposition of bacterial growth could lead to production of H_2S, CO_2, and NH_3. These gases, if trapped in the soil media could lead to an increased value of

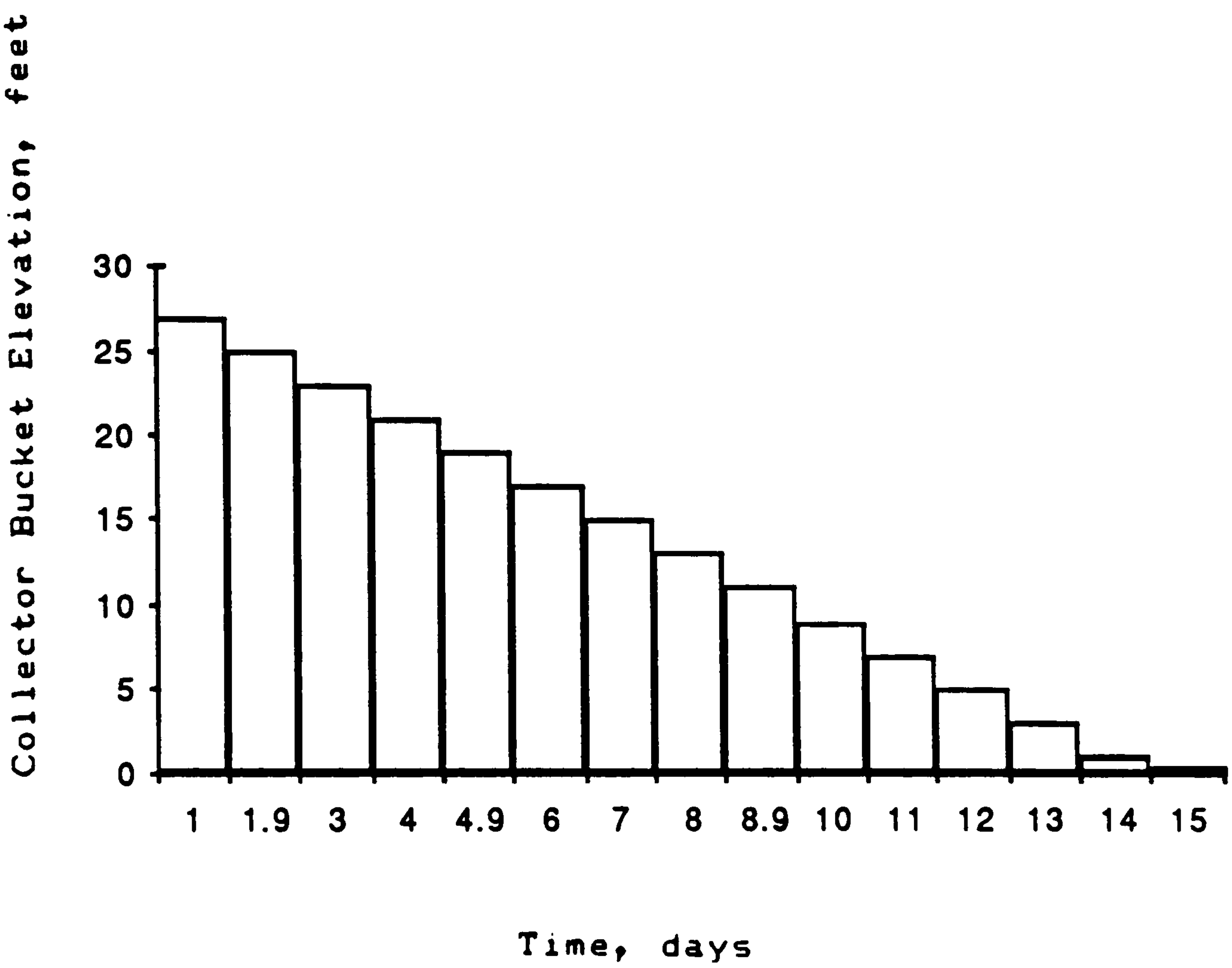


Figure 5.1 Time Variation of Collector Bucket Elevation

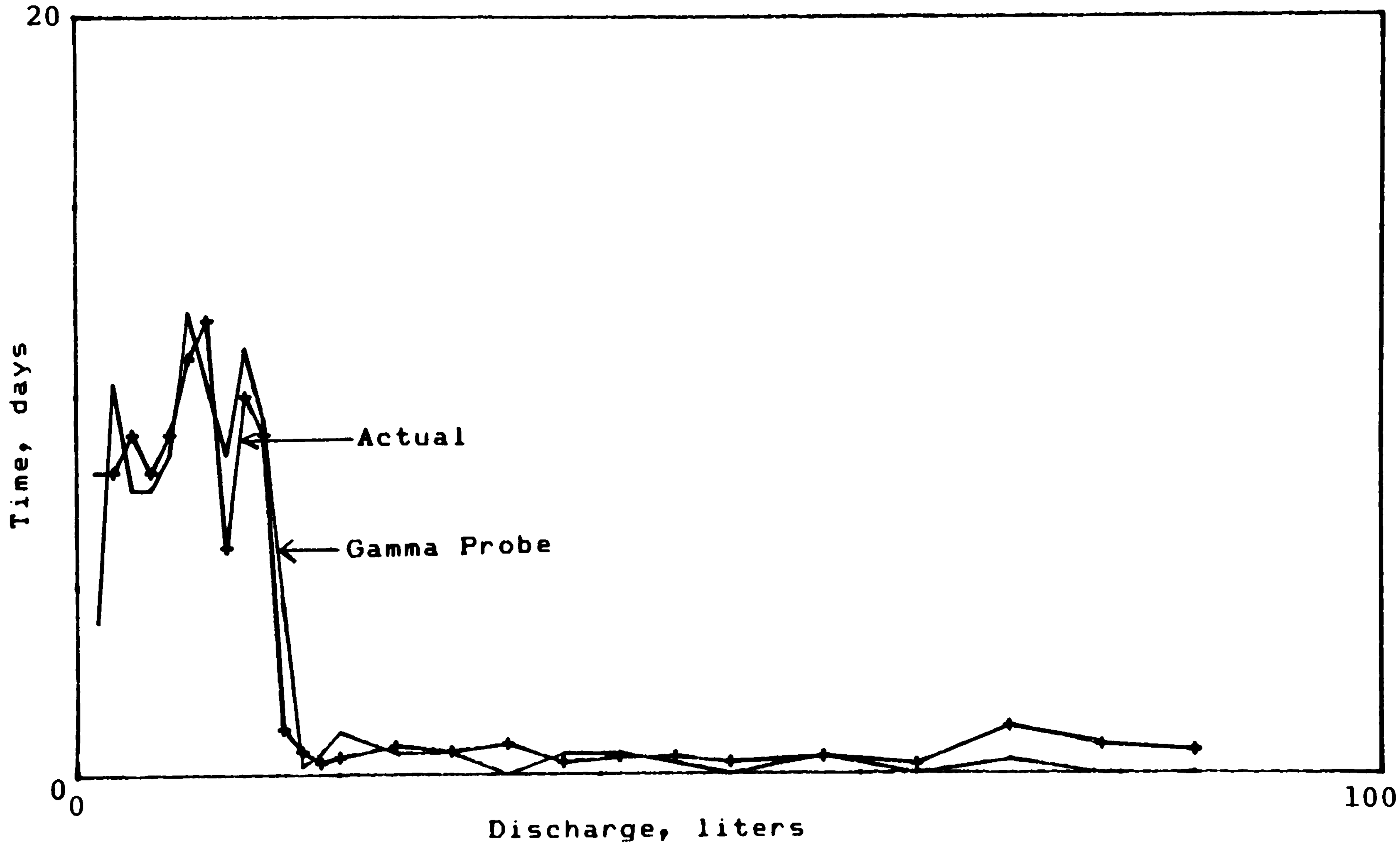


Figure 5.2 Time Variation of Discharge (Actual and Probe)

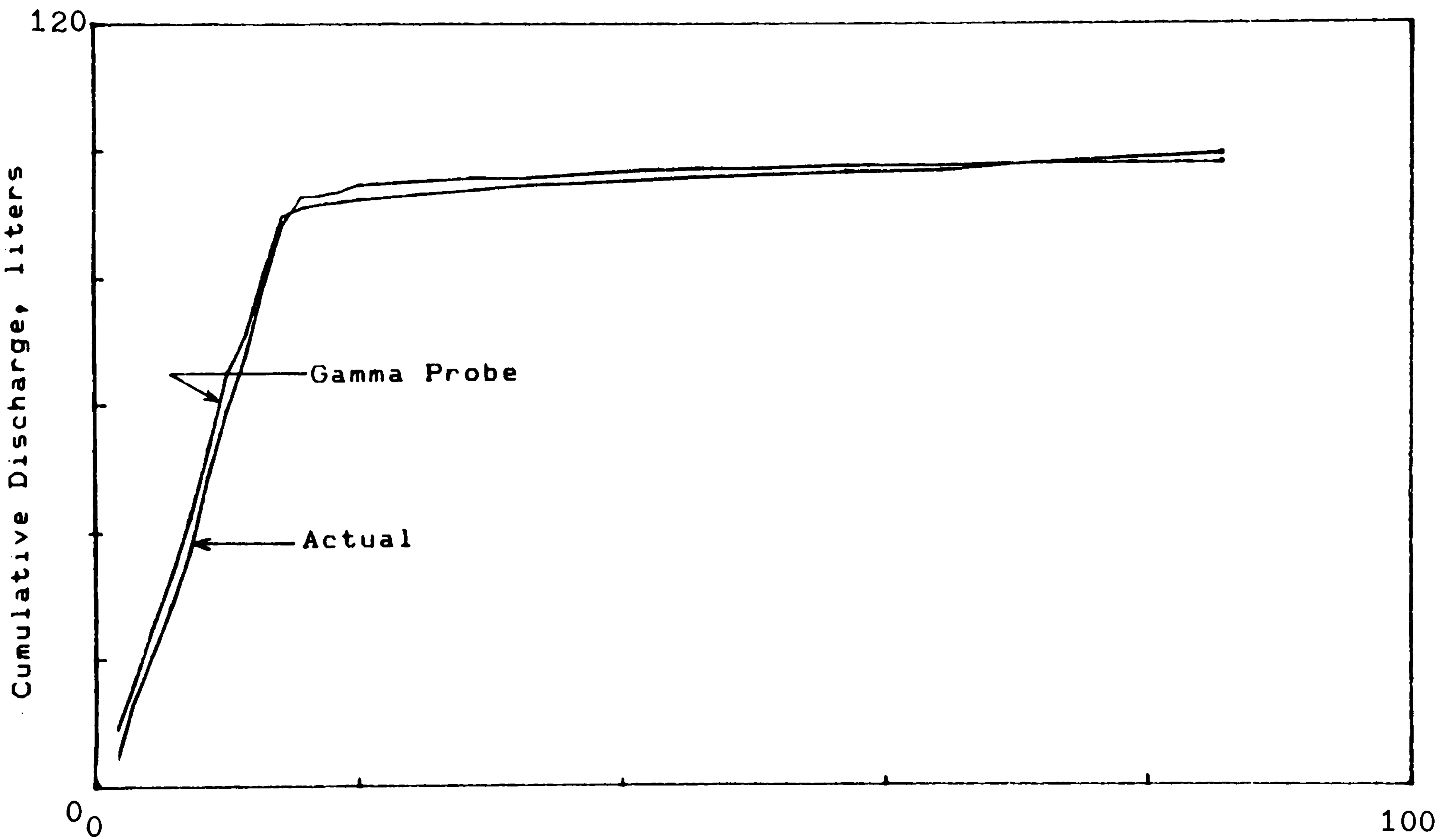
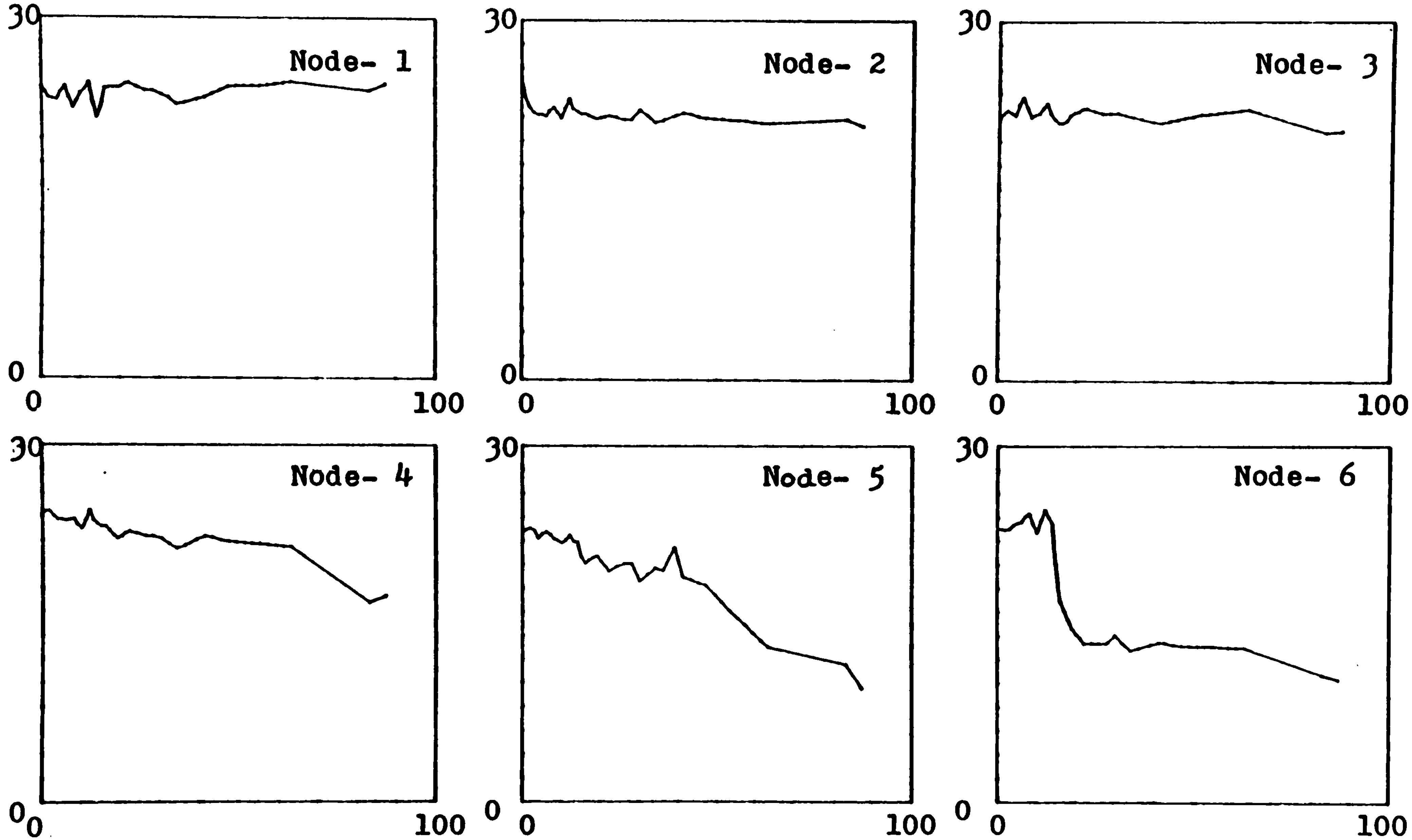
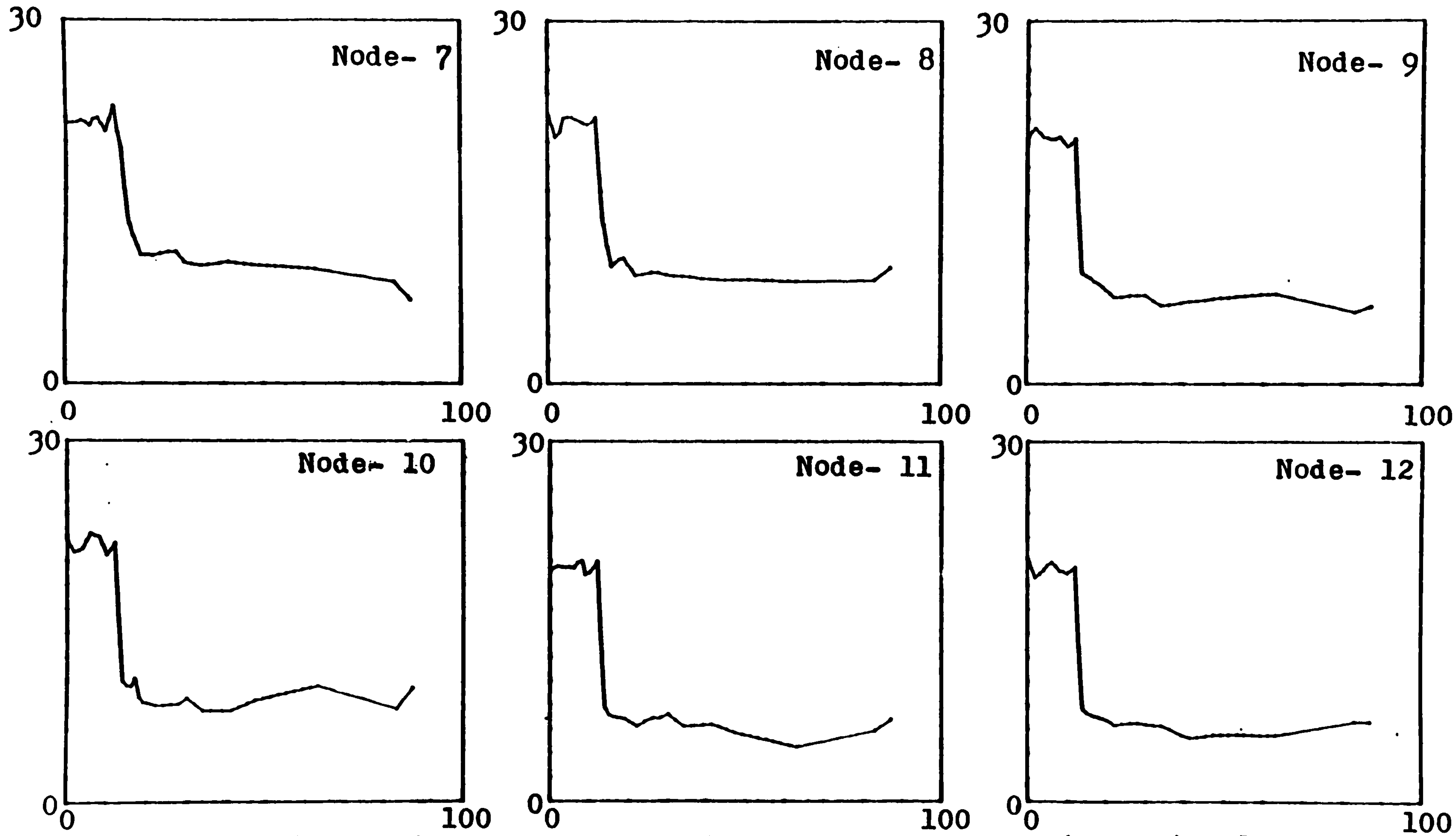


Figure 5.3 Cumulative Discharge versus Time (Actual and Probe)



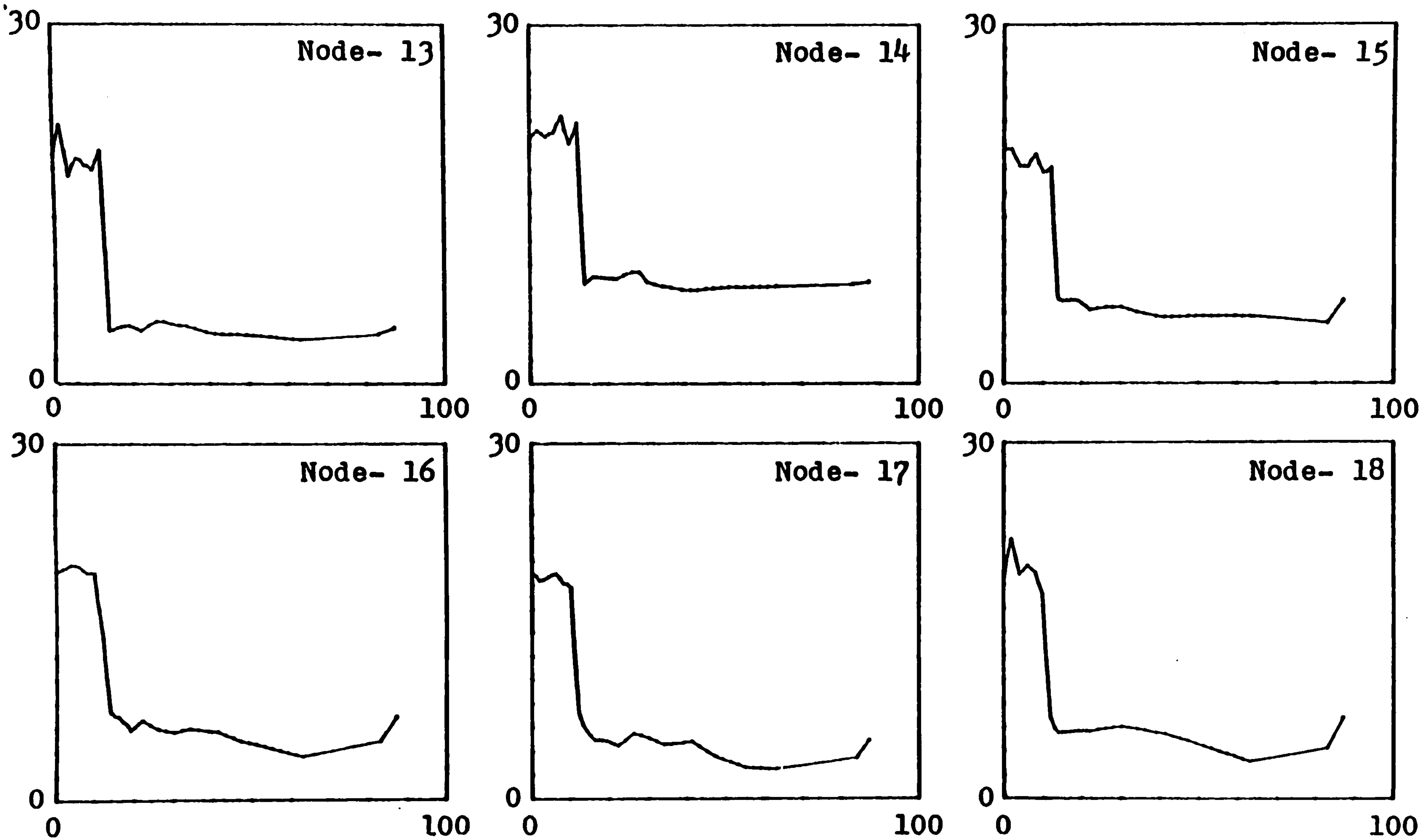
Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.4 Time Variation of Water Content, Node- 1 to 6



Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.5 Time Variation of Water Content, Node- 7 to 12



Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.6 Time Variation of Water Content, Node- 13 to 18

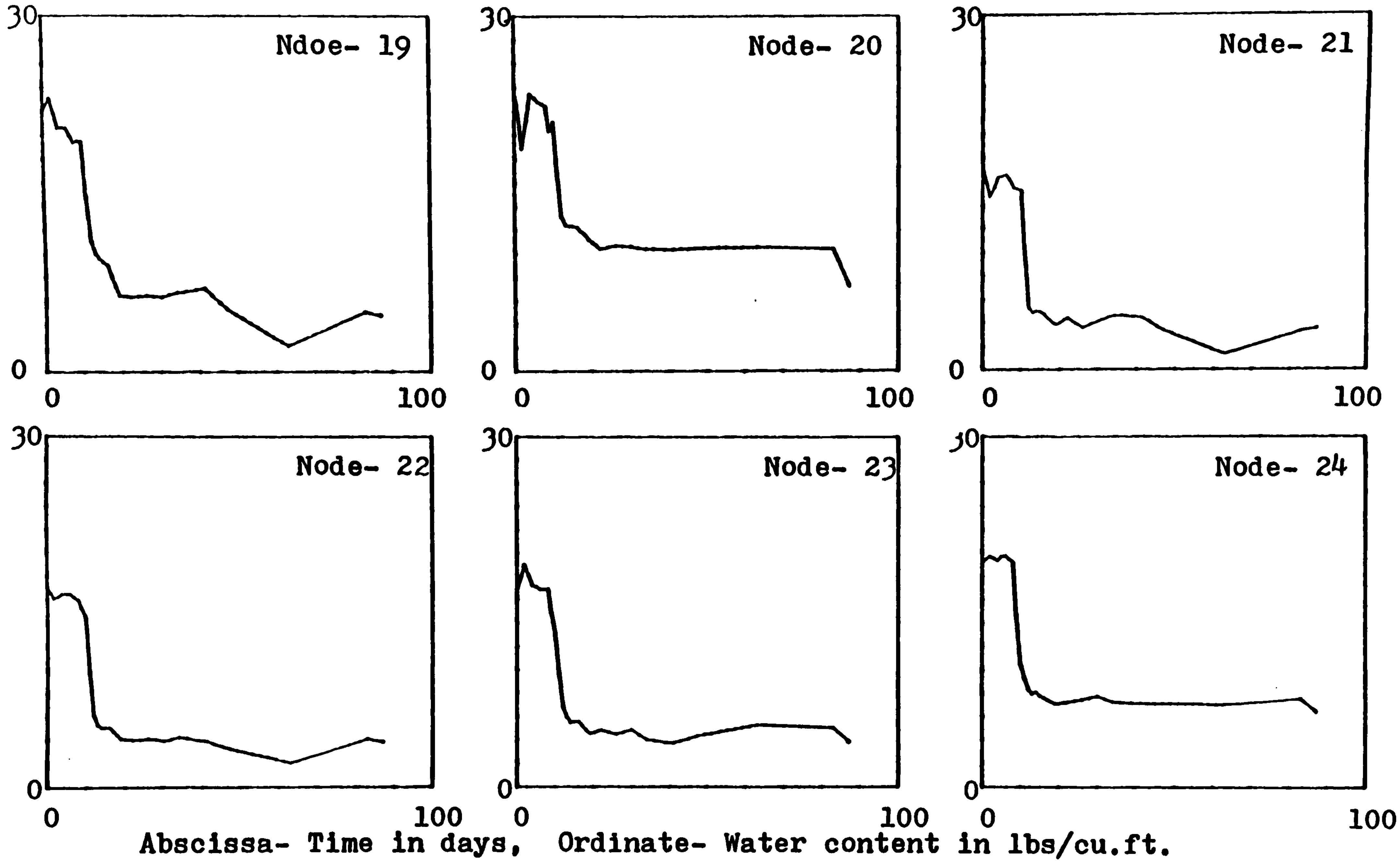
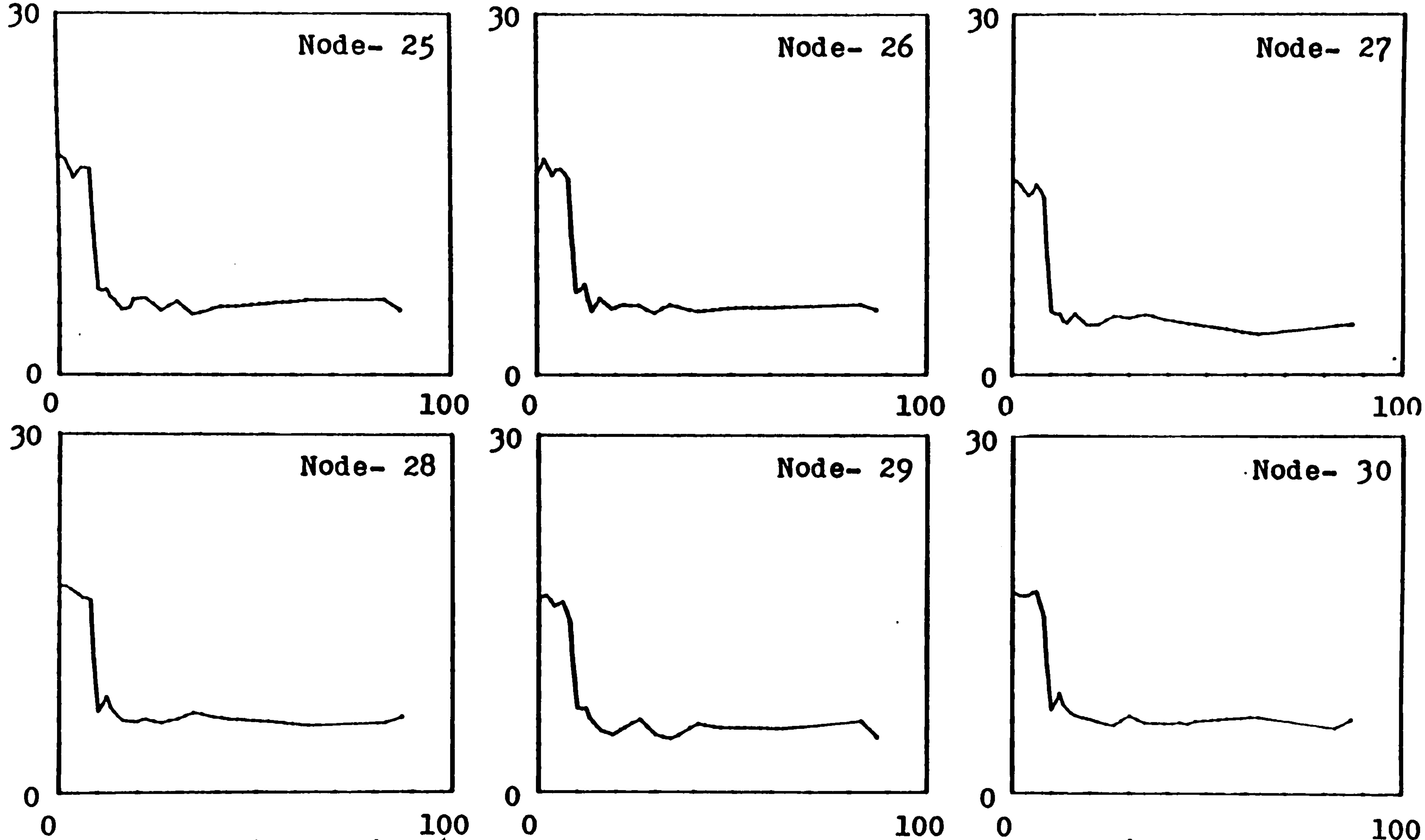
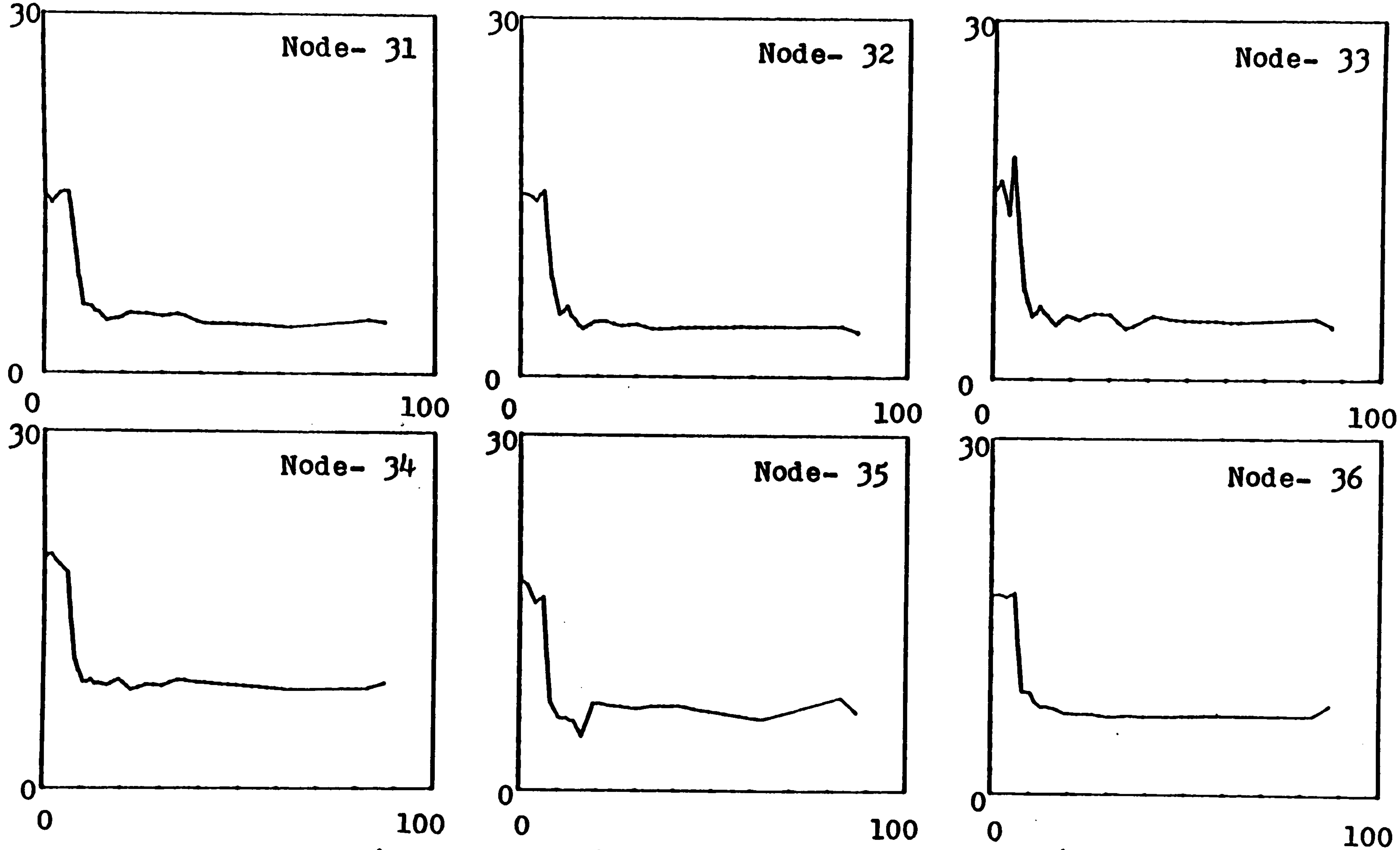


Figure 5.7 Time Variation of Water Content, Node- 19 to 24



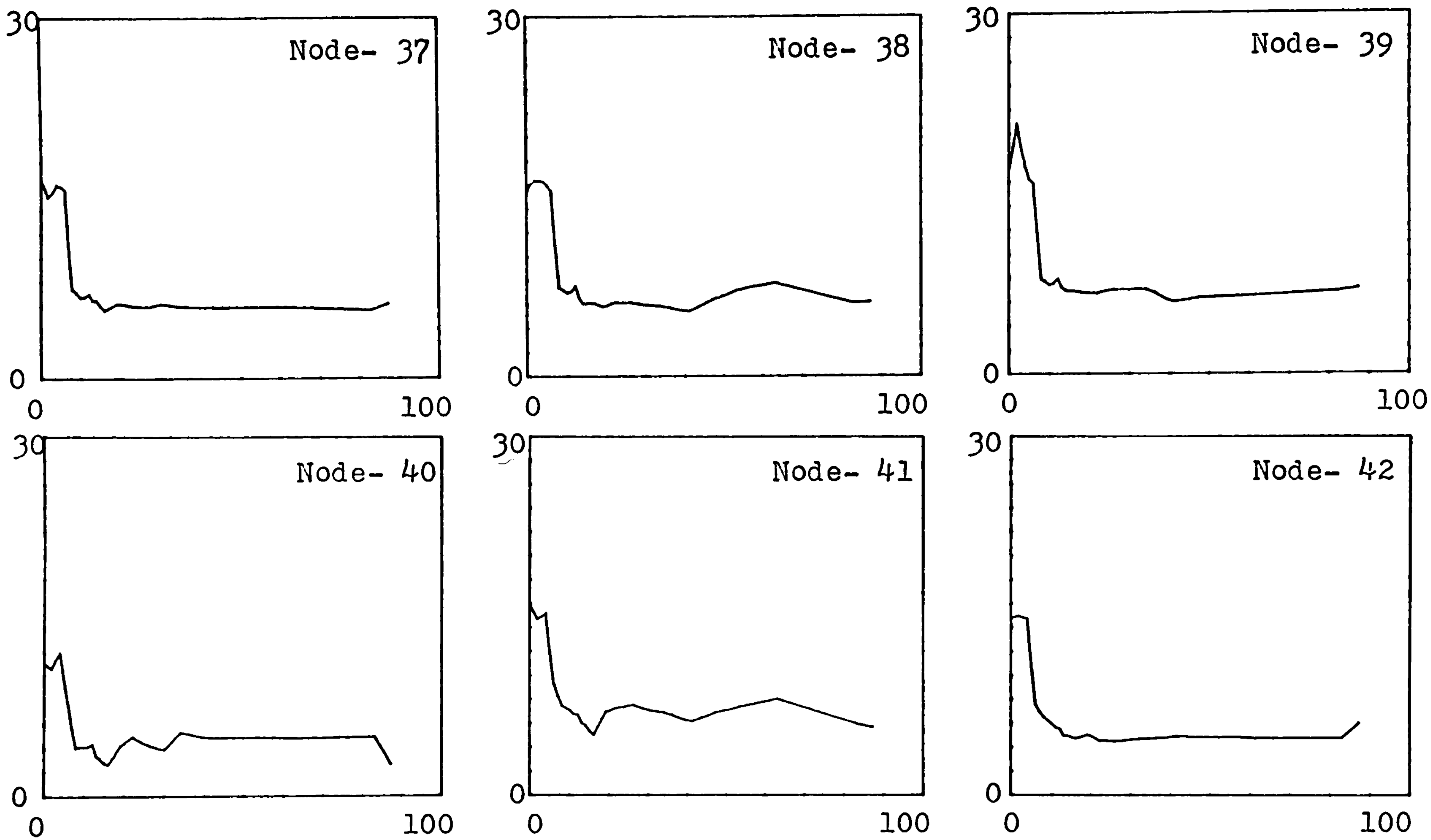
Abscissa- Time in days, Ordanate- Water content in lbs/cu.ft.

Figure 5.8 Time Variation of Water Content, Node- 25 to 30



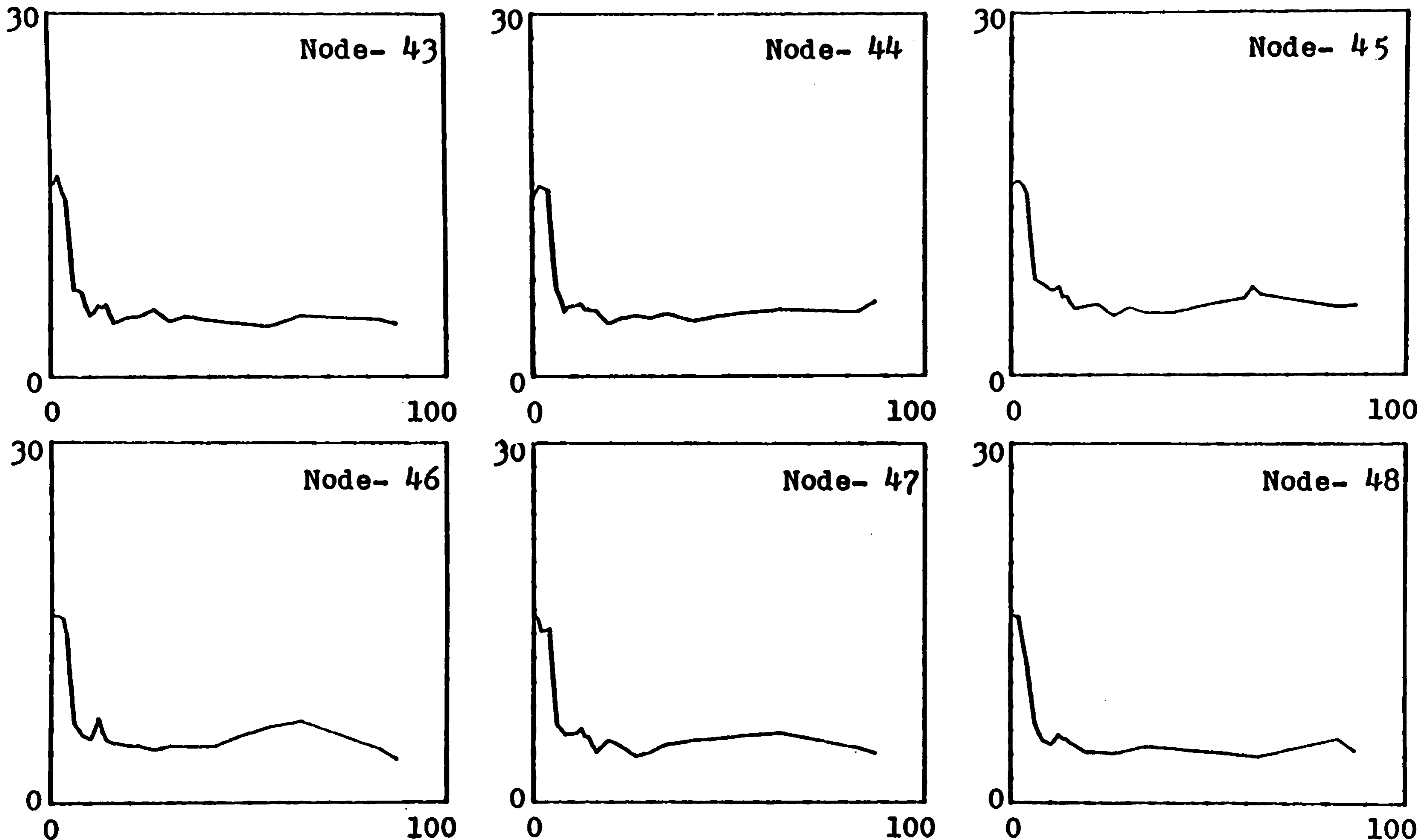
Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.9 Time Variation of Water Content, Node- 31 to 36



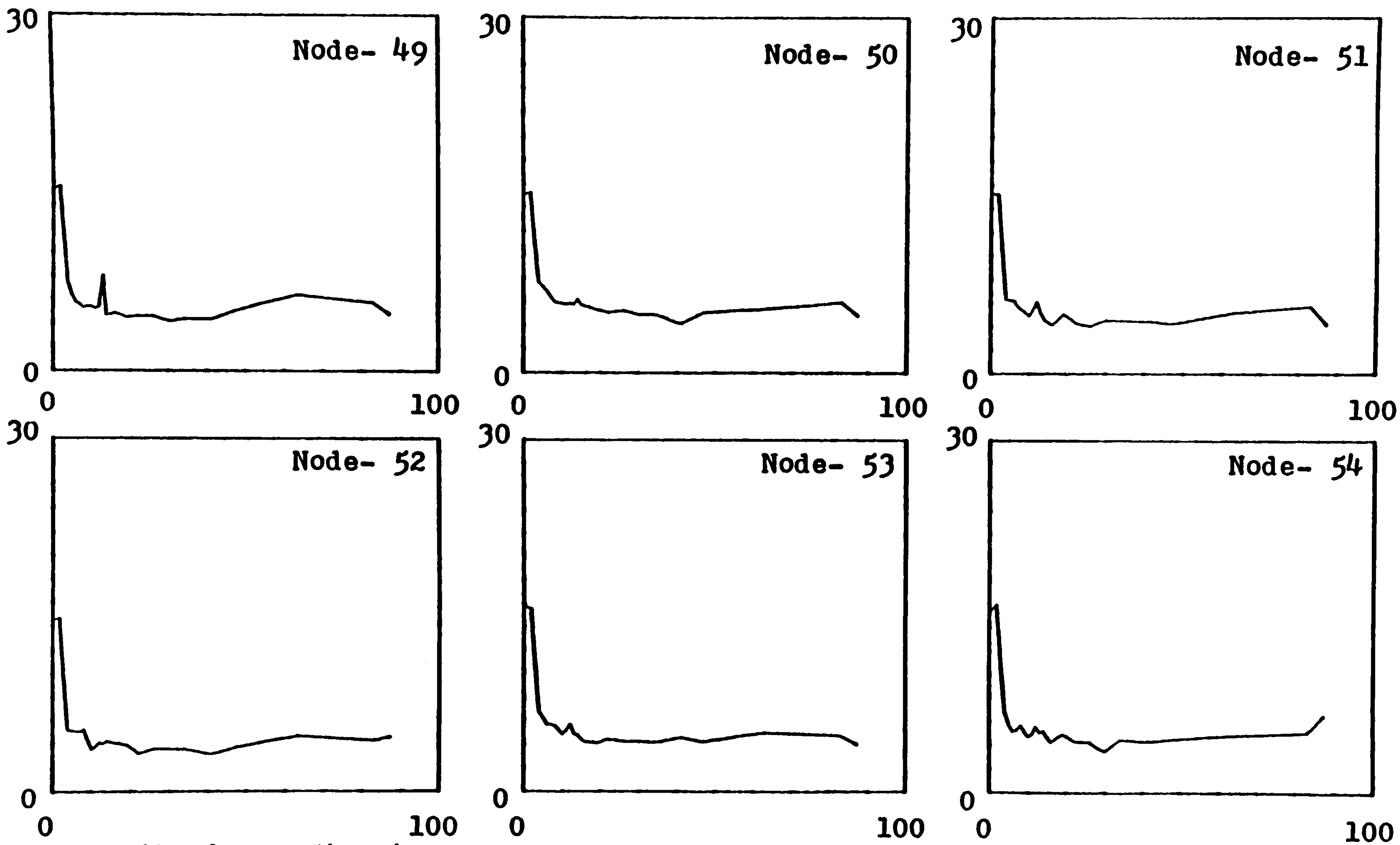
Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.10 Time Variation of Water Content, Node- 37 to 42



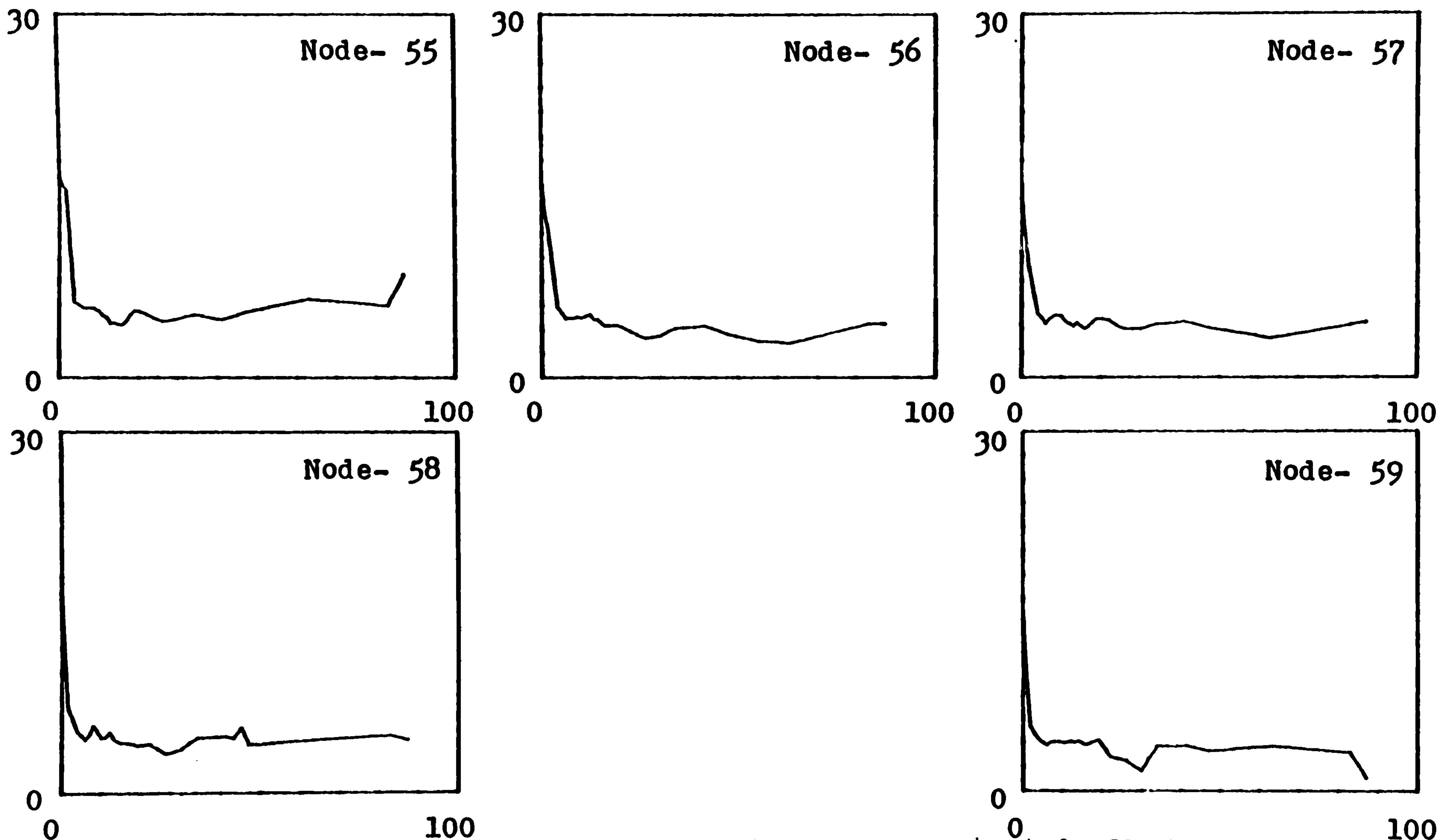
Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.11 Time Variation of Water Content, Node- 43 to 48



Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.12 Time Variation of Water Content, Node- 49 to 54



Abscissa- Time in days, Ordinate- Water content in lbs/cu.ft.

Figure 5.13 Time Variation of Water Content, Node- 55 to 59

porosity in the lower portions of the column (Metcalf and Eddy, Inc., 1972). The column was treated with calcium hypochlorite solution. Also, it is possible that the subsequent upward movement of the water table may have transported some biological residue thus reducing the porosity of the upper zones. Excessive rust from the interior surface of the column, fine particles of inorganic soil materials, and organic matter originally present in the soil could have caused porosity changes.

2) Air trapped in the sand column could be a factor leading to lower values of saturation and porosity. Although efforts were made to avoid trapping of air by applying a vacuum to the top of the column, air binding in the interstices could occur.

3) The sudden decrease in water content at a node is related to the time when the boundary conditions at the bottom of the column were approximately equal to the elevation of the node. These phenomenons were evident in the time versus water content plots (see Figure 5.4 to Figure 5.13). A change in the elevation of the bucket causes a sudden increase in the total hydraulic gradient. An increased value of the total hydraulic gradient reduces the value of the water content in the region around the collector bucket position until near equilibrium conditions are attained. This phenomenon is repeated until the bucket has reached the bottom of the column.

4) A sudden increase or decrease in water content values can also be attributed to the inadequate stablization of the Troxler gamma probe or improper instrumentation and operation.

5) The lack of a better correlation between the measured volumes of daily discharge and the computed value of drainage from the column using the gamma probe readings is questionable and can be attributed to biological growth in the column and instrumentation. A least square analysis of actual discharge and the discharge values obtained using the gamma probe data was performed and the values of discharge obtained using gamma probe were modified. A new value of constant was found for computing the density readings from the gamma count. A comparison between the two discharge data's was made (see Figure 5.3).

Computer Results

Input variables to the computer program have been described in Chapter IV. Some of the main features of the column drainage simulation using the computer program are;

1) The soil properties of K(h) versus h and θ versus h discussed in Chapter III were modified slightly to yield results closer to the experimental values. Several combinations of these soil characteristic curves were

tried and the curves yielding the closest approximation to the measured outflow have been adopted for analysis purposes. Further modifications were found to have minor effects on the total volume of discharge from the column.

2) The computer program was used to simulate the drainage conditions in the experimental system. The elevation of the collector bucket was reduced in the same pattern as in the experimental system. Similar time periods were used for simulation. After the collector bucket reached the bottom of the column, a 25-hour simulation time period was used.

3) Values of 0.75 for w and 0.45 for alpha were found to yield reasonably good results without numerical instability problems. Several combinations of these parameters were tried. Smaller values of alpha and of the time, step, Dt were necessary to calculate the potential values immediately after the drop in the elevation of the collector bucket. The program was found to be very sensitive to such shock changes in the boundary conditions. Higher values of alpha and time step Dt during such changes led to convergence problems.

Simulation Procedure

To start the program, the elevation of the collector bucket, ZD, was set at 30 feet (i.e., the top of the column). The value of the pressure-potential was calculated at each of the nodes distributed uniformly along the height of the column as the initial, pressure-potential values. The initial pressure-potential values were used to compute the initial-saturation values from the soil characteristic data using the cubic-spline interpolation subroutine. The saturation values were integrated along the height of the column to obtain the volume of water available in the sand column under saturated conditions. The elevation of the collector bucket was reduced by 3 feet similar to the experimental system. The initial estimate of the end-of-time-step values of H was based on a steady state head distribution.

The program simulates the drainage process for the time period until the next change in elevation of the collector bucket. The program output consists of the pressure potential values at 60 nodes. These pressure potential values are used to obtain saturation values using the cubic spline subroutine. The saturation values are integrated along the height of the column to calculate the volume of water available in the column at the end of time period. The difference between begining and ending volumes gives the volume of water drained during the time period.

The elevation of the bucket was decreased in accordance with the experiment. A similar drop in the elevation and in time periods were used until the bucket had reached an elevation of 0.5 feet from the bottom of the column. The volumes of water were accumulated over the time periods to give the total volume of water drained at the end of a certain number of days. Thereafter, the bucket elevation was maintained at a constant level and a time period of 25 hours used for computation of pressure potential values. The program indicates that the column ceases to drain at the end of 18 days while a very small volume (60 ml) of water was collected from the experimental system for the same time interval.

A plot of the time variation of discharge is shown in Figure 5.14, and a comparison with the actual experimental results is shown in Figure 5.15. A comparision of cummlative discharge has been done in Figure 5.16. The following observations can be made from Figure 5.16:

1) The volume of water collected after 18 days of drainage was found to be 92.15 liters using the program while a volume of 90.97 liters was actually collected from the experimental setup.

2) The column ceases to drain after 18 days while in the actual test system, the column continues to drain although the volume measured is very small, about 40 ml.

3) The volume of water collected after about 70 days is 96.80 liters which is 5 percent more than the volume indicated by the program.

4) The discharge values computed by the program show an excellent correlation to the actual discharge values available from the drainage experiment for steady flow conditions.

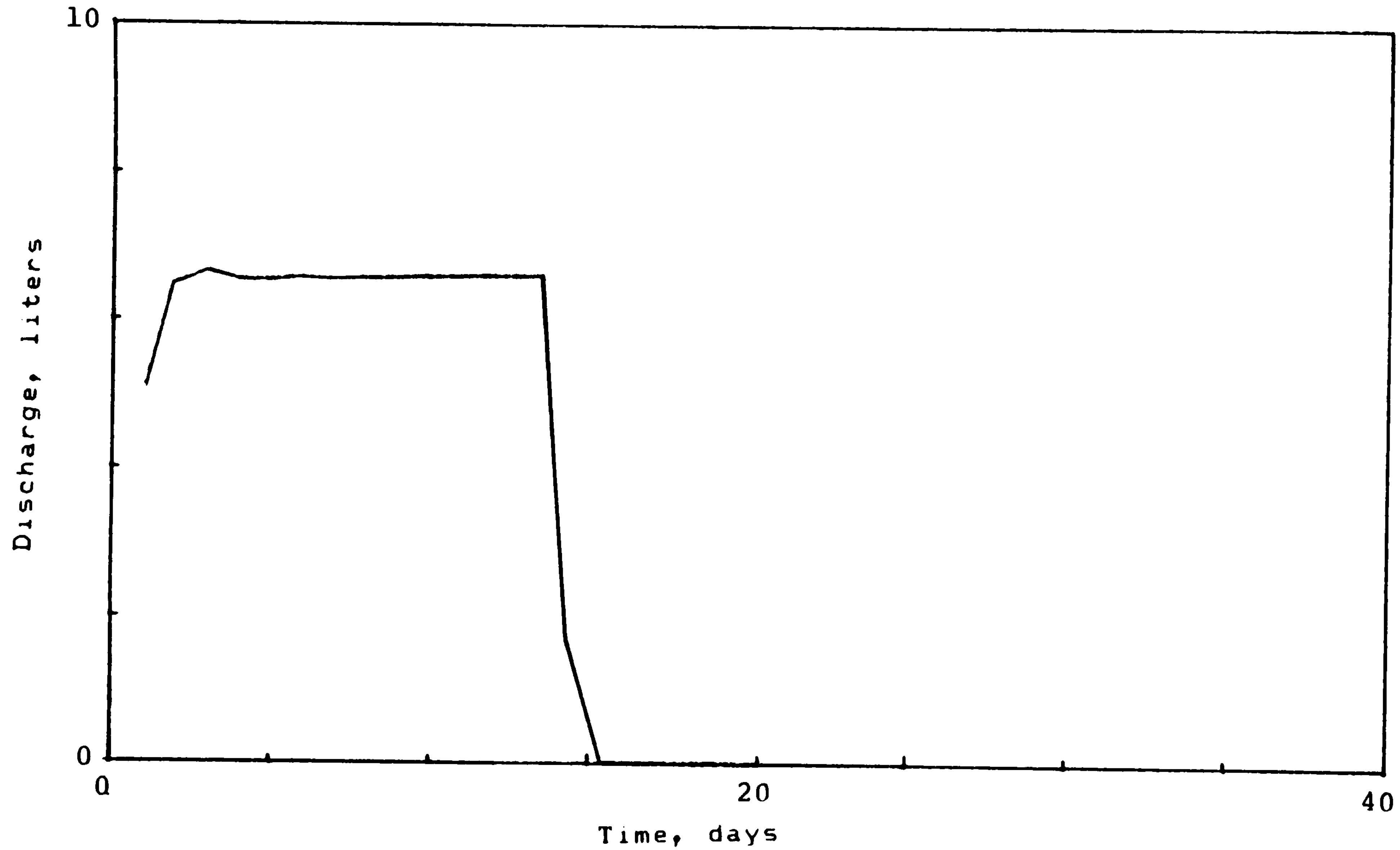


Figure 5.14 Time Variation of Discharge (Program)

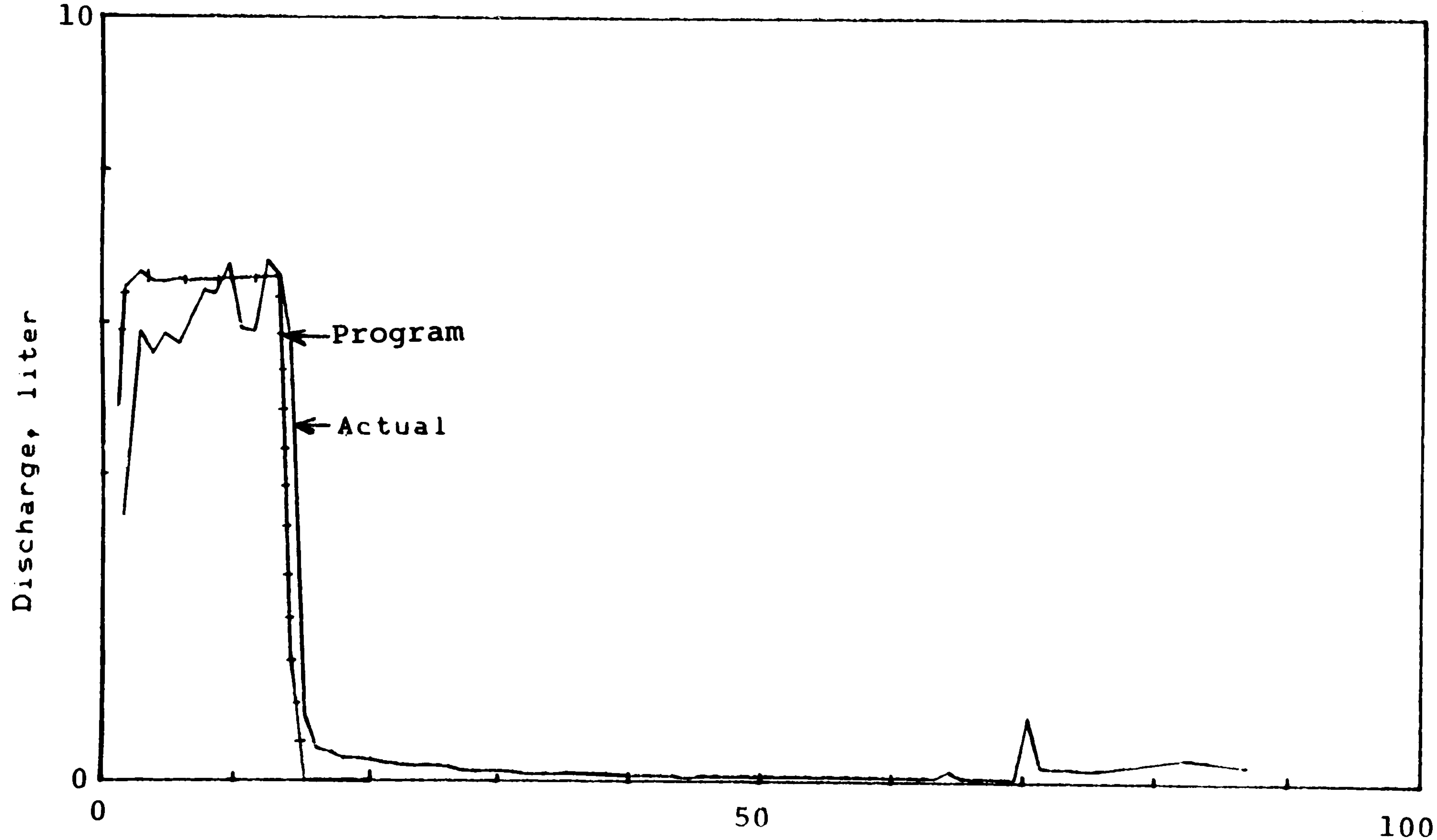


Figure 5.15 Comparison of Discharge

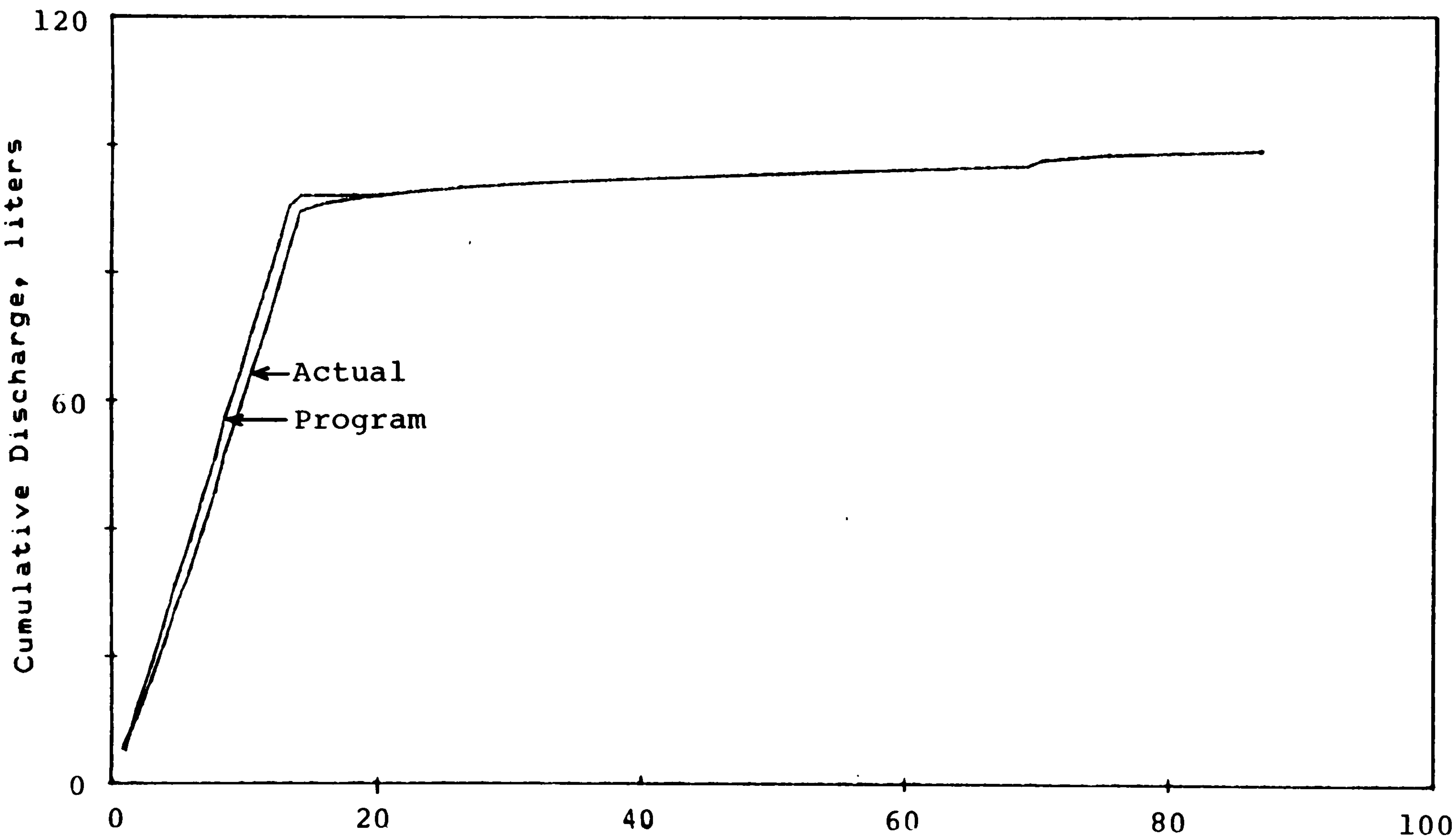


Figure 5.16 Comparision of Cumulative Discharge

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this chapter is to review the initial objectives set forth in Chapter I and to discuss each objective as it pertains to the results of this research. The final segment of this chapter is devoted to recommendations to improve upon the application of these findings and to assist in the resolution of the concern that brought about this study: the validity of Darcy's law for flow in unsaturated media.

Conclusions

The first objective established was to collect experimental data (water content and pressure potential) using a 30-ft sand column. It was not possible to collect pressure potential data for transient as well as steady state flow conditions because of experimental problems; thus, only the discharge and water content data were used. It was assumed that the entire sand column was saturated uniformly (i.e., $\theta_{sat} = 0.365$). However, experimental data indicates that saturation varied from 0.20 at the top to 0.385 at the bottom of the sand column. As the water table was lowered in small steps, the upper portion of the column drained rapidly until no further reduction in water content was detected. Thus, the sand lost water by draining to the water table. Water lost due to evaporation was negligible as the temperature variation in the test shaft was small during the period of drainage. Drainage was relatively rapid at the start when the water content and the hydraulic conductivities were large. As water content and the resulting conductivities decreased, drainage rates declined.

The second objective of this study was to develop a mathematical model using the finite-difference method. The Crank-Nicholson formulation has been successfully used to derive a nonlinear, second degree, finite-difference model for vertical drainage in a sand column. The model is used to simulate drainage conditions using an implicit solution technique. However, the model is found to be very sensitive to sudden changes in the boundary condition. These changes are brought about by the drop in the elevation of the collector bucket which produces a transient flow condition. The value of time step (Dt) and alpha were kept small to incorporate the effects of changes in boundary conditions. Smaller values of Dt and alpha lead to numerical stability. The values of Dt and alpha were reduced for a certain number of time steps; (i.e., 10); thereafter the values were increased as discussed in Chapter V.

The fourth objective of the study was to predict the time for the drainage to cease. The experimental data indicate that discharge had not ceased at the end of the test. A discharge of 60 ml was measured on the 90th day and drainage continued after that interval. The program predicts a time of 18 days which is less than the experimental results have indicated. It is possible to obtain such results because the actual soil characteristic curves were not available for simulation purposes. Extrapolation of the available soil characteristic data lead to smaller values of K and θ for higher values of matric potential.

The last objective was to test the validity of Darcy's law for flow of water in unsaturated media. The validity of Darcy's law can be tested by using either steady-state or transient-state flow systems as discussed in Chapter I. Drainage under gravity was considered as a transient flow until the discharge was almost negligible. Thereafter, for all practical purposes, the flow was considered steady. The discharge distribution predicted by the model was very close to the experimental data for transient flow conditions. Thus, Darcy's law is valid for the transient flow conditions. However, the validity of Darcy's law for steady-state flow systems depend on the final distribution of the matric potential. The model predicted that the matric potential distribution remained the same after the drainage had ceased. Experimental data for the matric potential distribution for steady state flow is not available, therefore, the model results cannot be justified. Matric potential values less than vapour pressure value of 0.59 ft would lead to vapourization of water in the soil media. This would lead to the discontinuity of water; thus, the derivatives of total potential cannot be obtained. It is not possible to justify the validity of Darcy's law for flow in unsaturated media as the necessary experimental data is not available from the current drainage tests.

Recommendations

Although the research does not provides a definite answer to the validity the Darcy's law for flow in unsaturated media, it does have future applications.

The collection of the experimental data for the matric potential in the long column drainage test is essential. The design of the manometer systems should be improved.

Precautions should be taken to prevent biological growth in the experimental column. Biological growth can be devastating and leads to erroneous readings.

Care should be taken to insure that the air is expelled in the first wetting cycle. A vacuum should be applied to the top while wetting the column.

Mathematical models should be expanded to include the equation of vapour movements in the unsaturated zone of the column.

LIST OF REFERENCES

Bear, Jacob. Dynamics of Flow Through Porous Media New York: American Elseveir, 1972.

Blair, P. M., and Weinaug C. F. "Solution of Two-Phase Flow Problems Using Implicit Difference Equations." Paper (SPE 2185), American Institute of Mining, Metallurgical and Petroleum Engineers Inc., Annual Fall Meeting, September 29 - October 2, 1968: 417-424.

Bower, Herman. Groundwater Hydrology New York: McGraw Hill, 1978.

--------- "Unsaturated Flow in Ground Water Hydraulics" Journal of Hydraulics Division ASCE Vol. 5 1964: 121-143.

Coleman, E. A., "A Laboratory Study of Lysimeter Drainage Under Controlled Soil-Moisture Tension." Soil Science Vol. 62 1946: 365-387.

Day, P. R., and Luthin, J. N., "A Numerical Solution of the Differential Equation of Flow for a Vertical Drainage Problem." Soil Science Society of America Proceedings Vol. 20 1956: 443-447.

Eckis, R., and Gross, P. L. K., "South Coastal Basin Investigation- Geology and Groundwater Storage Capacity of Valley Fill: California Division." Water Resources Bulletin Vol. 45 1934: 273-.

Elrick, D. E., "Unsaturated Flow Properties of Soils." Australian Journal of Soil Reserve Vol. 1 1962: 1-8.

Freeze, R. A., "The Mechanism of Natural Ground-water Recharge and Discharge. 1) One-Dimensional, Vertical, Unsteady, Unsaturated Flow above a Recharging or Discharging Groundwater Flow System." Water Resources Research Vol. 5 No. 1 February 1969: 153-171.

Green, W. H., and Ampt, G. A., "Studies on Soil Physics. Part I- The Flow of Air and Water Through Soils." Vol. 4 1911: 1-24.

Hanks, R. J., and Bowers, S. A., "Numerical Solution of the Moisture Flow Equation for Infiltration into Layered Soils." Soil Science Society Proceedings 1962: 530-534.

Hazen, A., "Experiments Upon the Purification of Sewage and Water at the Lawrence Experimental Station, No. 1" Massachusetts State Board Health Publications No. 34 1892: 425-633.

Hillel, Daniel. Introduction to Soil Physics Orlando: Academic Press, 1982.

Hornberger, G., and Remson, I., "A Moving Boundary Model of a One-Dimensional Saturated-Unsaturated, Transient Porous Flow System." Water Resources Research Vol. 6 No. 3 1979: 896-905.

Jeppson, Roland W., "Determination of Hydraulic Conductivity-Capillary Pressure Relationship from Saturation-Capillary Pressure Data from Soils." PRWG 59c-4 1970: 1-20.

Johnson, A. I., Morris, D. A., and Prill, R. C., "Specific Yield and Related Properties- an Annotated Bibliography." U. S. Geological Survey Open File Report No. 2 1961: 245-.

Johnson, A. I., Prill, R. C., and Morris, D. A., "Specific Yield- Column Drainage and Centrifuge Moisture Content." U. S. Geological Survey Water-Supply Paper No.1662-A 1963: 60-.

King, F. H., "Principles and Conditions of the Movement of Groundwater" U. S. Geological Survey 19th Annual Report Part-2 1899: 58-91.

Klute, A., "A Numerical Method for Solving the Flow Equation for Water in Unsaturated Materials." Soil Science Vol. 73 1952: 105-116.

Lamb, T. W., "Capillary Phenomenon in Cohensionless Soils." American Society of Civil Engineers Transactions Vol. 116 1951: 401-494

Laverett, M. G., "Capillary Behavior of Porous Solids" American Institute of Mining and Metallurgical Engineers Transactions Vol. 1942 1941: 152-168.

Lebedeff, A. F., "The Movement of Ground and Soil Waters" International Congress Soil Science Proceedings Vol. 1 1927: 459-494.

Liakopoulos, Aristides C., "Theoretical Aspects of the Flow of Water Through Anisotropic Unsaturated Soils." Bulletin of the International Association of Scientific Hydrology 1960: 62-70.

Luthin, J. N., and Miller, R. D., "Pressure Distribution in Soil Columns Draining into Atmosphere." Soil Science Society of America Proceedings Vol. 17 1953: 329-333.

Marx, J. W., "Determining Gravity Drainage Characteristics on the Centrifuge." American Institute of Mechanical Engineers, Petroleum Transactions Vol. 207 1956: 88-91.

Metcalf and Eddy, Inc. Water Engineering- Collection Treatment Disposal New York: McGraw Hill, 1972.

Miller, E. E., and Miller, R. D., "Theory of Capillary Flow: I. Practical Implications., and II. Experimental Information." Soil Science Society of America 1954: 267-275.

Morel-Seytoux, Hubert J., and Billica, Judith A., "A Two Phase Numerical Model for Prediction of Infiltration: Application to a Semi-Infinite Soil Column." Water Resources Research Vol. 21 No. 4 1985: 607-615.

Piper, A. M., "Geology and Ground Water Hydrology of the Mokelumns Area, California." U. S. Geological Survey Water Supply Paper No. 780 1939: 101-122.

Rawlins, Stephen L., and Gardner, Walter H., "A Test of the Validity of the Diffusion Equation for Unsaturated Flow of Soil Water." Soil Science Society Proceedings 1963: 507-511.

Remson, I., Fungaroli, A. A., and Hornberger, G. A., "Numerical Analysis of Soil-Moisture Systems." Journal of Irrigation and Drainage Proceedings ASCE NO. 3 1967: 153-165.

Richard, L. A., "Capillary Conduction of Liquids Through Porous Mediums." Physics Vol. 1 1931: 318-333.

Rose, Walter D., "Some Problems Connected With The Use of Classical Description of Fluid/Fluid Displacement Processes." Industrial and Engineering Chemistry Vol. 62 1962: 229-240.

Rubin, J., "Theoretical Analysis of Two-Dimensional, Transient Flow of Water in Unsaturated and Partly Unsaturated Soils." Soil Science Society of America Proceedings Vol. 32 No. 5 1968: 607-611.

Scott, V. H., and Corey, A. T., "Pressure Distribution During Steady Flow in Unsaturated Sands." Soil Science Society Proceedings 1961: 270-273.

Smith, W. O., "Infiltration and Drainage in Uniform Sands: Mechanism of Gravity Drainage and its Relation to Specific Yield of Uniform Sands." U. S. Geological Survey Professional Paper 402-A 1961: A1-A12.

Stahl, R. F., Martin, W. A., and Huntington, R. L., "Gravitational Drainage from Unconsolidated Wilcox Sand." American Institute of Mining Metallurgical Engineers Transactions Vol. 151 1943: 138-146.

Stearn, H. T., Robinson, T. W., and Taylor, G. H., "Geology and Water Resources of the Mokelumne Area, California" U. S. Geological Survey Water Supply Paper No. 619 1930: 151-172.

Sposito, G., "General Criteria for the Validity of the Buckingham-Darcy Flow Equation." Soil Science Society of America Vol. 44 1980: 1159-1168.

Swartzendruber, Dale, "Modification of Darcy's Law for the Flow of Water in Soils." Purdue University Agricultural Experimentation Station Journal Paper No. 1814 1961: 22-29.

----------------, "Non-Darcy Flow Behavior in Liquid-Saturated Porous Media." Journal of Geophysical Research Vol. 67 No. 13 1962: 5205-5213.

Terwillinger, P. L., et al., "An Experimental and Theoretical Investigation of Gravity Drainage Performance." American Institute of Mining Metallurgical Engineers Transactions Vol. 192 1951: 285-296.

Watson, K. K., "Experimental and Numerical Study of Column Drainage." _Journal of the Hydraulic Division_, ASCE Vol. 93 1967: 1-15.

Whistler, F. D., and Klute, A., "The Numerical Analysis of Infiltration, Considering Hysteresis, into a Vertical Soil Column at Equilibrium Under Gravity." _Soil Science Society of America_ Vol. 29 No. 5 1965: 489-494.

White, W. N., "A Method of Estimating Ground-water Supplies Based on Discharge by Plants and Evaporation from Soils." _U. S. Geological Survey Water Supply Paper_ No. 659-A 1932: 106.

APPENDIX

COMPUTER PROGRAM

Computer program used to simulate vertical drainage conditions in a soil column using soil characteristic curves.

```
1000  MAXERR = 0.01 : FACT = 0 : SUM = 0
1010  INPUT "DAY =";DAY : IF DAY > 0 THEN 1050
1020  INPUT "ENTER NUMBER OF EQUALLY SPACED NODES "; NODE
1030  GOTO 1090
1040  REM.....READ COLUMN DATA.....
1050  PRINT"DAY=";DAY
1060  OPEN "SET.DAT" FOR INPUT AS FILE #3
1070  INPUT#3,NODE,DTI,W,ALPHA,DZ,L,ZD,DI,VOL,FACT
1080  CLOSE#3
1090  DEFDBL A-H,P-Z
1100  '.....INPUT SIMULATION TIME.....
1110  INPUT "FACT1 =";FACT1
1120  INPUT "FACT2 =";FACT2
1130  DIM A(300),B(300),C(300),E(300),K(75),TH(75),
          H(301,2)
1140  DIM Y(75),D2K(75),D2T(75),ANS(300),D2(75),YP(75)
1150  DIM DTDH1(300),M(300,1),HI(75),X(75),DKDH1(300),
          K1(300)
1160 '.....READ SOIL DATA AND COMPUTE 2ND DERIVATIVES.....
1170  OPEN "PODMNM3.DAT" FOR INPUT AS FILE #1
1180  FOR I4 = 1 TO 54
1190      INPUT #1, HI(I4), TH(I4)
1200      X(I4) = HI(I4) : Y(I4) = TH(I4)
1210   NEXT I4
1220  CLOSE#1
1230  N = I4
1240  GOSUB 2660
1250  FOR I = 1 TO N
1260        D2T(I) = ANS(I)
1270   NEXT I
1280  N1 = 0
1290  OPEN "SODMNM1.DAT" FOR INPUT AS FILE #2
1300  FOR N1 =1 TO 54
1310      INPUT #2, HI(N1),K(N1)
1320      Y(N1) = K(N1)
1330   NEXT N1
1340  CLOSE#2
1350  N = N1
```

```
1360  GOSUB 2660
1370  FOR I = 1 TO N
1380      D2K(I) = ANS(I)
1390    NEXT I
1400  IF DAY > 0 THEN 1560
1410  '.....ENTER COLUMN DATA.....
1420  INPUT "ENTER OMEGA, THE WEIGHTING VALUE ";W
1430  INPUT "ENTER LENGTH OF COLUMN (FT) ";L
1440  INPUT "ENTER DIA OF COLUMN (IN) ";DI
1450  INPUT "ENTER CONVERGENCE FACTOR, ALPHA ";ALPHA
1460  INPUT "INCREMENTAL DROP OF BUCKET,L1 (FT) ";L1
1470  INPUT "ENTER TIME STEP (SEC) ";DT
1480  VOL = 22/28*(DI/12)^2*L*TH(N) : DZ=L/NODE : ZD=L-L1
1490  '.....ESTABLISH INITIAL CONDITIONS .....
1500  DTI = DT
1510  FOR I = 1 TO NODE
1520      H(I,2) = ZD-ZD/L*(I-.5)*DZ
1530    NEXT I
1540  FOR I = 1 TO N : X(I) = HI(I) : NEXT I
1550  GOTO 1700
1560  '.....MODIFY CONVERGENCE FACTOR.....
1570  INPUT"INCREMENTAL DROP OF BUCKET,L1 (FT)";L1
1580  INPUT"NEW VALUE FOR CONVERGENCE FACTOR, ALPHA";ALPHA
1590  INPUT"NEW VALUE FOR DT=";DTI
1600  ZD = ZD-L1
1610  DT = DTI
1620  '.....READ POTENTIAL VALUES.....
1630  PRINT"DAY = ";DAY
1640  INPUT" ENTER NAME OF LAST OUTPUT FILE, LAST(DAY)";F$
1650  OPEN F$ FOR INPUT AS FILE #5
1660  FOR I = 1 TO NODE
1670  INPUT #5,H(I,2)
1680  NEXT I
1690  CLOSE#5
1700  '.....BEGIN ITERATION FOR TIME STEP, START WITH END
1710  '.....OF TIME STEP = BEGINNING OF TIME STEP VALUES.
1720  TIME1 = 0 : JJ=0
1730  JJ = JJ+1
1740  TIME1 = TIME1+DT
1750  ITER = 0
1760  FOR I = 1 TO NODE
1770      H(I,1) = H(I,2)
1780   NEXT I
1790  FOR J = 1 TO N: Y(J) = K(J): YP(J) = D2K(J): NEXT J
1800  FOR I = 1 TO NODE
1810  IF H(I,1) => X(N) THEN K1(I) = W*K(N) : DKDH1(I)
                                      = 0.00001 : GOTO 1830
1820  X1 = H(I,1) : GOSUB 2970 : K1(I) = W*Y1 : DKDH1(I) =
                                                      W*DYDX
1830    NEXT I
1840  FOR I = 1 TO N: Y(J) = TH(J): YP(J) = D2T(J): NEXT J
```

```
1850  FOR I = 1 TO NODE
1860  IF H(I,1) => X(N) THEN DTDH1(I) = 0.05 : GOTO 1880
1870      X1 = H(I,1) : GOSUB 2970 : DTDH1(I) = W*DYDX
1880    NEXT I
1890  FOR I = 1 TO NODE
1900      H(0,2) = 2*ZD-H(1,2) : H(0,1) = 2*ZD-H(1,1)
1910      H(NODE+1,2) = H(NODE,2)-DZ : H(NODE+1,1) =
                                              H(NODE,1)-DZ
1920  IF X(N) <= H(I,2) THEN DKDH = 0.00001 : DTDH = 0.05

                               : PERM = K(N) : GOTO 2010
1930  ' IF TRUE, THE NODE IS SATURATED
1940  CP = H(I,2) : X1 = CP
1950  FOR J = 1 TO N :Y(J) = TH(J) :YP(J) = D2T(J) :NEXT J
1960  GOSUB 2970 : REM FIND DTDH FOR CURRENT VALUE OF CP
1970  DTDH = DYDX : THETA = Y1
1980  FOR J = 1 TO N : Y(J) = K(J) :YP(J) = D2K(J) :NEXT J
1990  GOSUB 2970 : REM FIND DK/DH AND K FOR CURRENT VALUE
                                                     OF CP
2000  DKDH = DYDX : PERM = Y1
2010           B1 = DKDH1(I)+(1-W)*DKDH
2020           B2 = K1(I)+(1-W)*PERM
2030           B3 = DTDH1(I)+(1-W)*DTDH
2040   A(I) = -B1/B3/4/(DZ^2)*(-2*W*H(I+1,1)
               +2*W^2*H(I+1,1)2*W*H(I-1,1))
2050   A(I) = A(I)-B1/B3/4/(DZ^2)*(-2*W^2*H(I-1,1)
               +H(I-1,2)-2*W*H(I-1,2)+W^2*H(I-1,2))
2060   A(I) = A(I)-B1/B3/4/(DZ^2)*(-2*DZ+2*W*DZ
               +(-1+2*W-W^2)*H(I+1,2)+4*B2*W/B1)
2070   B(I) = 1/DT+8*B2*W/B3/4/(DZ^2)
2080   C(I) = -B1/B3/4/(DZ^2)*(2*W*H(I+1,1)-2*W^2*H(I+1,1)
              -2*W*H(I-1,1)+2*W^2*H(I-1,1)+H(I+1,2))
2090   C(I) = C(I)-B1/B3/4/(DZ^2)*(-2*W*H(I+1,2)
              +W^2*H(I+1,2))
2100   C(I) = C(I)-B1/B3/4/(DZ^2)*(2*DZ-2*W*DZ
              +(-1+2*W-W^2)*H(I-1,2)+4*B2*W/B1)
2110   Q1 = B1/B3*(-W^2*H(I+1,1)+W^2*H(I-1,1)-2*W*DZ
                                      +4*B2*(1-W)/B1)
2120   Q2 = 4*B2*(2*W-2)/B3+4*DZ^2/DT
2130   Q3 = B1/B3*(W^2*H(I+1,1)-W^2*H(I-1,1)+2*W*DZ
                                     +4*B2*(1-W)/B1)
2140   E(I) = (Q1*H(I-1,1)+Q2*H(I,1)+Q3*H(I+1,1))/4/DZ^2
2150   '.....MODIFY BOUNDARY CONDITIONS.....
2160  IF I <> 1 THEN 2200
2170      B(1) = B(1)-A(1)
2180      E(1) = E(1)-2*ZD*A(1)
2190  GOTO 2230
2200  IF I <> NODE THEN 2230
2210      B(NODE) = B(NODE)+C(NODE)
2220      E(NODE) = E(NODE)+C(NODE)*DZ
2230  NEXT I
```

```
2240  N1 = N : N = NODE : GOSUB 3170 : N = N1
2250  '.....SOLVE TRIDIAGIONAL SET , RETURN ANSWERS IN
            ANS().....
2260  ERR1 = 0  : ENODE = 0
2270  FOR I = 1 TO NODE
2280  IF ABS(H(I,2)-ANS(I)) > ERR1 THEN ERR1 = ABS(H(I,2)
                                    -ANS(I)) : ENODE = I
2290  H(I,2) = ALPHA*ANS(I)+(1-ALPHA)*H(I,2)
2300  NEXT I
2310  ITER = ITER+1
2320  IF ERR1 > MAXERR THEN 1890
2330  IF MAXERR > 0.001 THEN MAXERR = MAXERR*0.01
2340  '.....CONVERGENCE ACHIEVED, RECORD RESULTS.....
2350  IF NOT JJ => 10 THEN 2650
2360  IF TIME1 => 1000 THEN DT = 500 ELSE DT = TIME1:JJ =1
2370  IF TIME1 => FACT1 THEN DT = 100
2380  TTIME = TIME1/3600
2390  IF RTTIME < FACT2 THEN 2650
2400  VOLT = 0 : TTIME = 0
2410  FOR J = 1 TO N : Y(J) = TH(J) : YP(J) = D2T(J) :
                                     DTDH = 0.05 : NEXT J
2420  FOR I = 1 TO NODE
2430  IF X(N) <= H(I,2) THEN M(I,1) = Y(N) : GOTO 2470
2440  IF X(1) >= H(I,2) THEN M(I,1) = Y(1) : GOTO 2470
2450  X1 = H(I,2) : GOSUB 2970
2460  M(I,1) =Y1
2470  VOLT = VOLT+(22/28*(DI/12)^2*DZ*M(I,1))
2480  NEXT I
2490  Q = VOL-VOLT
2500  SUM = Q
2510  '.....PRINT AND STORE RESULTS.....
2520  INPUT"LAST OUTPUT FILE NAME, LAST(DAY+1).DAT";B$
2530  OPEN B$ FOR OUTPUT AS FILE #4
2540  FACT = FACT+FACT2
2550  FOR I = 1 TO NODE
2560    PRINT #4,H(I,2),M(I,1)
2570    NEXT I
2580        PRINT #4,SUM
2590        PRINT #4,FACT
2600        PRINT #4,DAY
2610  CLOSE#4
2620  OPEN"SET.DAT" FOR OUTPUT AS FILE #3
2630    PRINT#3,NODE,DTI,W,ALPHA,DZ,L,ZD,DI,VOL,FACT
2640  CLOSE #3: IF DAY => 100 THEN 3340 ELSE 1010
2650  GOTO 1730
2660  '.....COMPUTE 2ND DERIVATIVES FOR CUBIC SPLINE.....
2670  I1 = I : ' PRESERVE I VALUE FROM MAIN PROGRAM
2680  FOR I = 2 TO N-1
2690      A(I-1) = X(I) - X(I-1)
2700      B(I-1) = 2*(X(I+1)-X(I-1))
2710      C(I-1) = X(I+1)-X(I)
```

```
2720      E(I-1) = 6*((Y(I+1)-Y(I))/(X(I+1)-X(I))
                    +(Y(I-1)-Y(I))/(X(I)-X(I-1)))
2730  NEXT I
2740  I2 = N : N = N-2 : ' PRESERVE N VALUE
2750  GOSUB 3170 : ' USE TRIDIGIONAL SUBROUTINE TO
                                                  FIND 2ND
2760  '                 DERIVATIVES, ANSWER IN ANS()
2770  N = I2
2780  '.....CHECK EACH INTERVAL FOR ZERO SLOPE POINTS.....
2790  ANS(N) = 0
2800  FOR I = N-1 TO 2 STEP -1
2810  ANS(I) = ANS(I-1)
2820  NEXT I
2830  ANS(1) = 0
2840  FOR I = 2 TO N
2850      A1 = ANS(I) - ANS(I-1)
2860      B1 = 2*(ANS(I-1)*X(I)-ANS(I)*X(I-1))
2870      C1 = -ANS(I-1)*X(I)^2+ANS(I)*X(I-1)^2-2*Y(I-1)
               +2*Y(I)+(ANS(I-1)-ANS(I))/3*(X(I)^2
               -2*X(I)*X(I-1)+X(I-1)^2)
2880  DISC = B1^2-4*A1*C1
2890  IF DISC <= 0 THEN 2940
2900      RT1 = (-B1+SQR(DISC))/2/A1
2910      RT2 = (-B1-SQR(DISC))/2/A1
2920  IF RT1 > X(I-1) AND RT1 < X(I) THEN
                PRINT "ONE MIN/MAX IN INTERVAL ";I-1
2930  IF RT2 > X(I-1) AND RT2 < X(I) THEN
                PRINT "ONE MIN/MAX IN INTERVAL ";I-1
2940  NEXT I
2950  I = I1
2960  RETURN
2970  '.....CUBIC SPLINE INTERPOLATION.....
2980  ORDER = 1
2990  IF X(N) > X(1) THEN GOTO 3040 : ' OR, REVERSE ORDER
                                                    OF X()
3000  ORDER = -1
3010  FOR ICS = 1 TO N/2
3020    X(ICS) = X(N+1-ICS)
3030  NEXT ICS
3040  JL = 1  : JH = N
3050  J1 = INT((JL+JH)/2)
3060  IF J1 = JL AND J1 => JH-1 THEN 3100
3070  IF X1 => X(J1) AND X1 < X(J1-1) THEN 3100
3080  IF X1 > X(J1) THEN JL = J1 : GOTO 3050
3090  JH = J1 : GOTO 3050
3100  Y1 = (YP(J1)*(X(J1+1)-X1)^3+YP(J1+1)*(X1-X(J1))^3)
            /6/(X(J1+1)-X(J1))+(Y(J1)/(X(J1+1)-X(J1))
            -(YP(J1)*(X(J1+1)-X(J1))/6))*(X(J1+1)-X1)
            +(Y(J1+1)/(X(J1+1)-X(J1))-(YP(J1+1)*(X(J1+1)
            -X(J1))/6))*(X1-X(J1))
3110  IF ORDER = 1 THEN 3150
```

```
3120  FOR ICS = 1 TO N/2
3130       X(ICS) = X(N+1-ICS)
3140  NEXT ICS
3150  DYDX = (-YP(J1)*(X(J1+1)-X1)^2+YP(J1+1)*(X1-
              X(J1))^2)/2/(X(J1+1)-X(J1))
              -((Y(J1)/(X(J1+1)-X(J1)))-(YP(J1)*(X(J1+1)-
                X(J1))/6))+((Y(J1+1)/(X(J1+1)
               -X(J1)))-(YP(J1+1)*(X(J1+1)-X(J1))/6))
3160  RETURN
3170  '.....SOLUTION FOR TRIDIAGIONAL SET OF
                                        EQUATIONS.....
3180  IT = I : ' PRESERVE VALUE OF I FROM CALLING PROGRAM
3190  FOR J = 1 TO N : ANS(J) = E(J) : NEXT J
3200  A(1) = 0 : C(N) = 0
3210  A(N) = A(N)/B(N)
3220  ANS(N) = ANS(N)/B(N)
3230  FOR I = 2 TO N
3240      J = N-I+2
3250      TEMP = 1/(B(J-1)-A(J)*C(J-1))
3260      A(J-1) = A(J-1)*TEMP
3270      ANS(J-1) = (ANS(J-1)-C(J-1)*ANS(J))*TEMP
3280  NEXT I
3290  FOR I = 2 TO N
3300  ANS(I) = ANS(I)-A(I)*ANS(I-1)
3310  NEXT I
3320  I = IT
3330  RETURN
3340  END
```

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